MATHEMATICS COMPETENCY EXAM STUDY GUIDE – PART A

2013

WAYNE STATE UNIVERSITY

ACADEMIC SUCCESS CENTER

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INTRODUCTION

Congratulations on taking the first step toward a successful math competency exam! This study guide is written as a resource to aid you as you prepare for Wayne State University's Math Competency Exam.

Math Placement

All students, including transfer and guest students, who plan to take MAT 0995, 1000, 1050, 1110, 1120, 1500, 1800 or 2010 as their first mathematics course at Wayne State University (WSU) must place into the course according to the policies of the WSU Department of Mathematics. Your placement into one of these courses will be based on your America College Test (ACT) Score. Please check with an academic advisor.

Any student who does not place into an MAT course using an ACT Math score as described above must take the WSU Mathematics Placement Examination to determine MAT course placement.

Math Competency

All educated individuals must show competency in their mathematical skills. These skills will help you to study other topics in which mathematics is a significant part of the subject matter, to deal with mathematical calculations that you might need to do in your prospective career, to manage your own personal finance, or to gain a better understanding of the mathematics that relate to public issues.

Break-down of the Mathematics Competency Exam

The examination consists of 55 questions. All students are given Part A (25 questions) and Part B (15 questions), but only those students who wish to place into MAT 2010 receive Part C (15 questions). You will be given 120 minutes to complete the exam. And remember there are no calculators allowed during the exam!

The following is a list of concepts by the different parts. This is by no means a complete list of all the concepts you would need to know.

PART A

- Add, subtract, multiply and divide rational numbers (includes integers, fractions, and decimals).
- Use the order of operations correctly to simplify expressions.
- Be familiar with introductory algebra skills, such as distributing and combining like terms.
- Simplify algebraic expressions.
- Be able to use exponent rules. Note that exponents may not be integers. For instance, it is possible to have a rational number or an algebraic expression as an exponent.
- Solve and graph linear equations.
- Solve literal equations for a given variable.
- Know geometry definitions such as the definition for an equilateral triangle and the radius of a circle.
- Solve introductory geometry problems. And be able to solve more complex problems using geometry definitions.

PART B

- Add, subtract, multiply and divide polynomials.
- Solve quadratic and rational equations.
- Solve and graph equations with inequalities.
- Solve and graph equations with an absolute value.
- Simplify radical expressions and solve radical equations.
- Know the definition, notation, and interpretation of functions.
- Understand and be able to solve problems with both rational and inverse functions.
- Understand and be able to solve problems with exponential and logarithmic functions. Also, be able to solve application problems with algebra, such as using logarithms to help determine the loudness of a sound.
- Solve problems with complex numbers.
- Know and be able to apply right triangle relationships, such the Pythagorean Theorem.
- Solve problems that involve parallel and perpendicular lines.

PART C

- Know and be able to use basic trigonometry definitions and identities.
- Use trigonometry to solve problems with triangles, such as finding the length of a side of a triangle using the sine function.
- Know the graphs of trigonometric functions.
- Understand the relationship between trigonometry and circles, including topics such as the unit circle and arc length.
- Be able to solve more complex problems involving parallel and perpendicular lines.

REMEMBER: This is not a complete list of concepts. This is a partial list made from observations of different mathematics competency exams!

Characteristics of a Successful Exam-Taker (students who have passed)

There are five common characteristics that we have noticed from those students who have passed the Mathematics Competency Exam.

1. They are dedicated to the preparation it takes to do well on the exam from the first day they begin their studies.
2. They almost never let a day pass by without working on mathematics, even if it is for 20 minutes here or there.
3. They seek outside help or extra tutoring on a weekly basis.
4. They take multiple practice exams under conditions that are similar to the actual test.
5. They allow themselves at least six weeks to prepare for the exam.

Notice that not one of these characteristics assumes someone is naturally good at math. In fact, that is a big myth! Those you may view as “math people” just have a stronger math background than you. None of us were born being able to solve algebraic story problems or graph linear inequalities. Those are skills we acquire over time when we make an honest effort to learn them.

Do you want to know the secret to being a good math student? The answer is **HARD WORK** and **PERSISTENCE**. Don’t give up! It might take you longer than other people, but you will eventually get it. Practice until you master a concept. Once you have it down cold, do it one more time. Only take the exam when you can honestly tell yourself that you’ve put in the time and effort.

Purpose of This Study Guide

The purpose of this study guide is to brush up on some of the topics that will help you improve your score on the mathematics competency exam. This study guide does not cover most of the material you will encounter on Part B and Part C of the exam. It is not a magic wand. **Reading through this study guide and neglecting to work the practice problems thoroughly will not enable you to do well on the competency exam.** You will have to work through certain sections more than once and work and re-work many of the practice problems multiple times.

This study guide is not intended to provide you with a comprehensive review of algebra. It will not make you a wizard in algebra. Do not think this is the end-all, be-all of mathematics. It is not even close. We are just scratching the surface of the math topics.

Lastly, there is usually more than one method that can be used to solve math problems. When you see a problem done in this study guide, it is because that was the way I know how to do it. If you already know a different way that you prefer to handling a certain problem, use it. All that matters to pass the competency exam is you end up with the same answer.

CHAPTER 1: NUMBERS AND NUMBER SETS

Number Sets

Whole Numbers	{0, 1, 2, 3, 4, 5, …} Numbers from 0 upwards without decimals
Counting Numbers	{1, 2, 3, 4, 5, …} Whole numbers from 1 upwards
Integers	{…, -3, -2, -1, 0, 1, 2, 3, …} All positive and negative counting numbers and zero
Rational Numbers	{$\frac{p}{q}$ where $q \neq 0$ and the decimal representation is terminating or repeating} Numbers that can be written as a fraction of integers and the decimal representation is terminating (stops at a place value) or repeating (place values start to repeat)
Irrational Numbers	{$\frac{p}{q}$ where $q \neq 0$ and the decimal representation is non-terminating and non-}repeating Any number that is not rational.
Imaginary Numbers	{Any number when squared gives a negative result} For example: $\sqrt{-4}$ NOTE: Imaginary unit is defined by $i = \sqrt{-1}$.
Complex Numbers	{a number that can be expressed as $a + bi$} NOTE: a and b are real numbers and i is the imaginary unit.

Vocabulary on Numbers

Odd Numbers	{…, -5, -3, -1, 1, 3, 5, …} Numbers that cannot be divided evenly into two groups
Even Numbers	{…, -4, -2, 0, 2, 4, …} Numbers that can be divided evenly into two groups __NOTE__: zero is an even number

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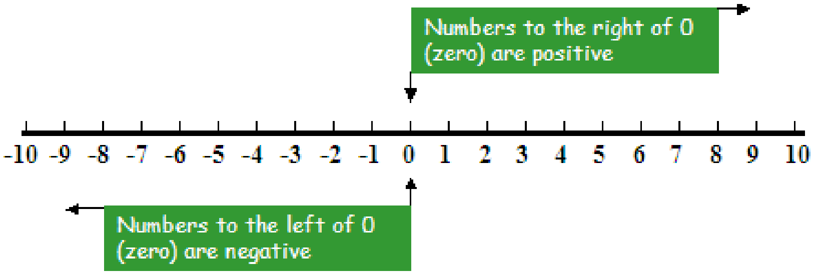
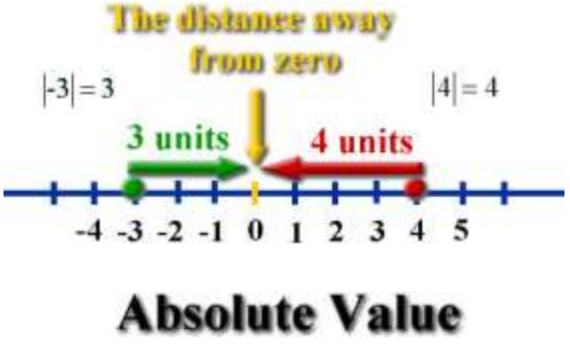
Positive numbers	{1, 2, 3, 4, 5, ….} All numbers (decimals, fractions and whole numbers) greater than 0
Negative numbers:	{All numbers (decimals, fractions and whole numbers) less than 0}

__NOTE__: zero is neither a positive number nor a negative number
Non-negative refers to numbers 0 or positive
Non-positive refers to numbers 0 or negative



Consecutive numbers	Numbers that follow each other in order (for example: 24, 25, 26)
Prime Numbers	{2, 3, 5, 7, 11, 13, 17, 19, 23, …} Whole numbers greater than 1 and are only divisible by 1 and itself. __NOTE__: 1 is not a prime number and 2 is the only even prime number.
Absolute value	Distance a number is away from zero on a number line. It refers to the non-negative value of the number.



CHAPTER 2: ESTIMATING

Many times you do not have to find the exact value of the answer. You can find an estimate and check choices for reasonableness. One technique to help you estimate is rounding.

Rules of Rounding

- Locate the digit in the place value to which you are rounding.
- Increase this digit by 1 if the next digit to the right is 5 or greater.
- Leave the digit unchanged if the next digit to the right is less than 5.

EXAMPLE 1

Round 4,582 to the indicated place value position.

a) The nearest ten b) to the nearest hundred c) to the nearest thousand

SOLUTION TO EXAMPLE 1

a) The digit in the tens place is 8. The number to the right is 2. So we leave the digit unchanged. So the answer is **<u>4,580</u>**.

b) The digit in the hundreds place is 5. The digit to the right is 8. So we increase our digit by 1 (from 5 to 6). Thus our answer is **<u>4,600</u>**.

c) The digit in the thousands place is 4. The digit to the right is 5. So we increase our digit by 1 (from 4 to 5). Thus our answer is **<u>5,000</u>**.

EXAMPLE 2

Find the actual answer. Use rounding to estimate the answer and compare to see if you actual answer is reasonable.

a) 3,234 + 2,578 b) 796 – 308 c) 93 x 198 d) 836 ÷ 22

SOLUTION TO EXAMPLE 2

a)
$$\begin{array}{rr} \text{Actual} & \text{Estimate} \\ 3{,}234 & 3{,}000 \\ \underline{+2{,}578} & \underline{+3{,}000} \\ 5{,}812 & 6{,}000 \end{array}$$

b)
$$\begin{array}{rr} \text{Actual} & \text{Estimate} \\ 796 & 800 \\ \underline{-308} & \underline{-300} \\ 488 & 500 \end{array}$$

c)
$$\begin{array}{rr} \text{Actual} & \text{Estimate} \\ 198 & 200 \\ \underline{\times 93} & \underline{\times 90} \\ 18{,}414 & 18{,}000 \end{array}$$

d)
$$\begin{array}{rr} \text{Actual} & \text{Estimate} \\ 38 & 40 \\ 22\overline{)836} & 20\overline{)800} \end{array}$$

<u>Test-Taking Strategy</u>: If the problem is a multiple choice, do you really need to find the exact answer? As a strategy, you can find the estimate and pick the answer that is closest to your estimate.

EXAMPLE 3

Friday's attendance at a baseball game was 21,904 people. The attendance at the baseball game on Saturday was 38,503.

a) What was the total amount of people who attended the baseball game on Friday and Saturday?

b) How many more people attended the baseball game on Saturday than on Friday?

SOLUTION TO EXAMPLE 3

Before we do the work, let's think about the problem and the type of problem. If this were a multiple choice question, I would just work out the estimate and look for the closest answer. But since it is not a multiple choice problem, I would need to show all work. I did both below.

a)	Actual	Estimate	Estimate 2	b)	Actual	Estimate	Estimate 2
	21,904	20,000	22,000		38,503	40,000	39,000
	+ 38,503	+ 40,000	+ 39,000		– 21,904	– 20,000	– 22,000
	60,407	62,000	61,000		16,599	20,000	17,000

NOTE: I did two different estimates for my answers. The first estimate I rounded to the nearest ten-thousands. For the second estimate, I rounded to the nearest thousand. Does it matter which estimate you use? Not really, but notice when we look at Estimate 2, it is a lot closer to the Actual answer. On a multiple choice question, if your answers are very close to each other, you would probably get a better answer when you round to the tens or hundreds.

TRY-THESE – Estimating

Round each of the following.

1. 684 to the nearest hundred
2. 8,237 to the nearest thousand
3. 1,539 to the nearest ten
4. 7,928 to the nearest hundred

Estimate. Then choose the letter of the actual answer.

		A	B	C
5.	27 × 32 × 49	423,360	4,233	42,336
6.	83,978 ÷ 398	21	211	441
7.	1,594 + 375 + 8,946	10,917	9,695	18,904

8. There are 12 rows of seat in an auditorium with 48 seats in each row.

   a) Approximately how many people will fit in the auditorium?

   b) There are 850 students that want to go to a concert in the auditorium. Will they all be able to be seated in the auditorium?

CHAPTER 3: ORDER OF OPERATIONS

In mathematics, the order that you do the adding, subtracting, multiplying, and/or dividing is important to getting the correct answer. Let's take a look at a simple problem.

$$4 + 3 \times 6$$

Student 1 might look at the problem and do "4 + 3 = 7 and 7 x 6 = 42. So the answer to the problem is 42!" Student 2 might do "3 x 6 = 18 and 4 + 18 = 22. So my answer is 22!"

Both students did all the work and got different answers. Then who has the correct answer? Mathematicians long ago came to this same dilemma. So they argued to who has the correct answer. Do you add first, and then multiply OR do you multiply first and then add? So they got together and came up with the Order of Operations.

In order to simplify an expression, you have to follow these steps in a certain order.

- First, perform all calculations within parentheses or other grouping symbols.
- Then, do all calculations involving exponents.
- Next, multiply OR divide in order from left to right.
- Lastly, add OR subtract in order from left to right.

Most of you might remember the word acronym "PEMDAS". I don't like referring to this acronym because people always confuse the order. Since Multiplication comes before Division in PEMDAS, they think that Multiplication always has to be done before Division. THIS IS WRONG!!! If we go back to the rules, it says "in order from left to right". If the multiplication is seen before division, multiply. If division is seen before the multiplication, divide.

EXAMPLE 1

Simplify using the order of operations: 12 ÷ 3 × 7 + 4

SOLUTION TO EXAMPLE 1

1.	There are no parentheses.	12 ÷ 3 × 7 + 4
2.	There are no exponents	12 ÷ 3 × 7 + 4
3.	Division comes first, so 12 ÷ 3 = 4.	4 × 7 + 4
4.	Multiplication is next. 4 × 7 = 28.	28 + 4
5.	Add the two numbers	32

EXAMPLE 2

Simplify using the order of operations: 5(20 ÷ 5 + 6) - 51

SOLUTION TO EXAMPLE 2

1.	Do the parentheses first. Notice there are two operations in the parentheses. So we need to do the division before adding in the parentheses. 20 ÷ 5 = 4	5(4 + 6) - 51
2.	There is still a parentheses so we need to do the operation in the parentheses. 4 + 6 = 10 **<u>NOTE</u>**: At this point we do not need the parentheses any more since there is just a value. But if we remove the parentheses, there would be two numbers without an operation.	5(10) - 51
3.	When a number is next to a parentheses, it is implied to be a multiplication.	5 × 10 - 51
4.	Multiplication is next. 5 × 10 = 50.	50 - 51
5.	Subtract. 50 - 51 = -1	-1

EXAMPLE 3

Simplify using the order of operations: $2(5-3(40 \div 10 \times 2+2)+1)^2$

SOLUTION TO EXAMPLE 3

1.	Before you go crazy, remember to break it down. We need to do the parentheses. Since there are two sets of parentheses, you simplify the inside parentheses before the outside. So we have to simplify the part that reads "$(40 \div 10 \times 2+2)$". Division comes before multiplication so divide. $40 \div 10 = 4$	$2(5-3(4 \times 2+2)+1)^2$
2.	Next comes the multiplication. $4 \times 2 = 8$	$2(5-3(8+2)+1)^2$
3.	Then add.	$2(5-3 \times (10)+1)^2$
4.	I still have the outer parentheses. So let's multiply. $3 \times 10 = 30$	$2(5-30+1)^2$
5.	Subtract. $5-30=-25$	$2(-25+1)^2$
6.	Add. $-25+1=-24$	$2(-24)^2$
7.	Now we have to square the parentheses. Since the negative sign is inside the parentheses, $(-24)^2=(-24) \times (-24)=576$	2×576
8.	Multiply to get the final answer.	1152

TRY THESE – Order of Operations

1. $3^3 + 5 \div 5 \times 6^2 - (-9)$

2. $\frac{1}{2} + 40 \div 2 + 5^2$

3. $5\big(3 - 6(4 + 2 \times 4)\big) + 2$

4. $2 - 3(4 - 4 \div 4 \times 2 + 2)^3$

5. $\left[\frac{1}{4} \div \frac{3}{4}\right]^2 + 3$

6. $\left[\frac{3}{5} \div \frac{3}{4} + \frac{1}{2}\right]^2$

CHAPTER 4: FRACTIONS

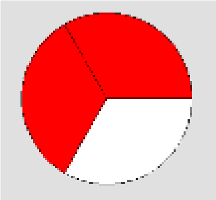
There are several operations you are expected to understand and perform on the competency exam. You will probably not see a problem that specifically asks you to add, subtract, multiply or divide two fractions. Instead you might encounter more difficult problems that include working with fractions (as in Problems 5 and 6 from Chapter 3: Try-These).

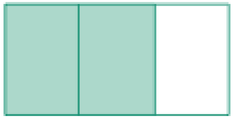
Let's think about some observations about fractions.

1. Fractions are basically the same as decimals.
2. Fractions are division problems in disguise.
3. Dividing and multiplying fractions are slight easier than adding and subtracting fractions.
4. When multiplying or dividing fractions, you do not need a common denominator.
5. **When adding or subtracting fractions, a common denominator is needed**.

Fractions represent equal parts of a whole. When you see a fraction like $\frac{2}{3}$, the number on top is called the **numerator**. It represents the number of parts being described. The number on the bottom is called the **denominator**. The denominator represents the total number of parts that make up one whole.

So $\frac{2}{3}$ means that there are 2 equal parts out of 3 parts. Visually, $\frac{2}{3}$ would look like





Equivalent Fractions

Any fractions that represent the same amount are called **equivalent fractions**. An equivalent fraction can be found by multiplying or dividing both the numerator and denominator by the same non-zero number.

EXAMPLE 1

Find four equivalent fractions of $\frac{2}{5}$.

SOLUTION TO EXAMPLE 1

$\frac{2\times 2}{5\times 2}=\frac{4}{10}$	$\frac{2\times 3}{5\times 3}=\frac{6}{15}$	$\frac{2\times 4}{5\times 4}=\frac{8}{20}$	$\frac{2\times 25}{5\times 25}=\frac{50}{125}$

<u>**NOTE**</u>: In the last solution, I used 25. I just wanted to show that we can multiply by any number we want to get an equivalent fraction.

The term "simplify" refers to reducing a fraction down to the lowest terms. When a fraction is simplified or reduced to lowest terms, there is no one number that can be evenly divided into *both* the numerator and denominator (other than 1).

EXAMPLE 2

Simplify $\frac{24}{42}$ to its lowest term.

SOLUTION TO EXAMPLE 2

Some of you might remember that you have to find the Greatest Common Factor (GCF) to divide the numerator and denominator. The GCF is great, but it not always the quickest to find the GCF. Just keep dividing out factors that you can divide into the numerator and denominator. It might take you a couple of extra steps, but you are still finding the answer.

$\frac{24 \div 2}{42 \div 2} = \frac{12}{21}$	$\frac{12 \div 3}{21 \div 3} = \frac{4}{7}$

Since there are no values that divide into 4 and into 7 evenly, we found our fraction in lowest terms.

The GCF of 24 and 42 is 6. So if we divide 6 from the numerator and denominator, we get the same answer.

$$\frac{24 \div 6}{42 \div 6} = \frac{4}{7}$$

<u>**NOTE**</u>: To save yourself some time when simplifying fractions, divide out larger numbers from the numerator and denominator.

No matter where you start, just remember that the exam's final answer will always be the most simplified.

Mixed Numbers and Improper Fractions

There are three types of fractions that can be written. The first is the **proper fraction**. These are the most common fractions that we see. A proper fraction is where the numerator is smaller than the denominator. Some examples of proper fractions are $\frac{1}{2}, \frac{3}{8}, \frac{10}{27}$, and $\frac{1000}{2589}$.

Another type of fraction is the **improper fraction**. An improper fraction has the numerator the same value or greater than the denominator. So a couple of examples of improper fractions are $\frac{3}{2}, \frac{12}{5}, \frac{15}{15}$, and $\frac{104}{26}$. Notice that even if the numerator and denominator are the same, it is still an improper fraction.

The last type of fraction is called the **mixed fraction** (or **mixed number**). A mixed fraction is a mixture of a whole number value and a fraction. A few examples of mixed numbers are $8\frac{1}{2}$, $10\frac{2}{7}$, and $93\frac{3}{4}$.

Converting Mixed Fractions into Improper Fractions

- Multiply the whole number part by the fraction's denominator.
- Add it to the numerator.
- Write that result on top of the original denominator

EXAMPLE 3

Write $2\frac{3}{10}$ as an improper fraction.

SOLUTION TO EXAMPLE 3

1.	Multiply the whole number (2) by the fraction's denominator (10).	$2 \times 10 = 20$
2.	Add it (20) to the numerator (3)	$20 + 3 = 23$
3.	Place over the original denominator (10).	$\frac{23}{10}$

Converting Improper Fractions to Mixed Numbers

- Divide the numerator by the denominator.
- Write down the whole number answer.
- Place the remainder over the original denominator.

EXAMPLE 4

Write $\frac{11}{4}$ as a mixed number.

SOLUTION TO EXAMPLE 4

1.	Divide numerator (11) by denominator (4).	$\begin{array}{r} 2 \\ 4\overline{)\,11} \\ -8 \\ \hline 3 \end{array}$
2.	Write down the whole number.	2
3.	Place the remainder (3) over the original denominator (4).	$2\frac{3}{4}$

Multiplying Fractions

Most people think we should always do adding or subtracting before multiplication or division. In the case of fractions, multiplying and dividing of fractions are slightly easier than adding and subtracting.

Before we get into how to multiply fractions, first we have to make sure all the numbers are written as fractions (either proper or improper fractions). We cannot have any mixed numbers. If you do not remember how to convert mixed numbers to improper fractions, please review the previous section of this chapter.

Now that we have all fractions, let's multiply our fractions. Multiply the numerators together. Then multiply the denominators together. And simplify the result if you can. That's it!

EXAMPLE 5

Multiply $\frac{3}{4} \times \frac{2}{6}$.

SOLUTION TO EXAMPLE 5

1.	Multiply the numerators.	$3 \times 2 = 6$
2.	Multiply the denominators.	$4 \times 6 = 24$
3.	Simplify the fraction if you can	$\frac{6 \div 6}{24 \div 6} = \frac{1}{4}$

EXAMPLE 6

Multiply $2\frac{1}{3} \times 4$.

SOLUTION TO EXAMPLE 6

1.	Make sure numbers are written as proper or improper fractions. You can change any whole number into an improper fraction by placing a 1 in the denominator.	$\frac{7}{3} \times \frac{4}{1}$
2.	Multiply numerators together.	$7 \times 4 = 28$
3.	Multiply denominators together.	$3 \times 1 = 3$
4.	Simplify the fraction if you can. Notice that the fraction is an improper fraction. Convert it to a mixed number.	$\frac{28}{3} = 9\frac{1}{3}$

Dividing Fractions

Dividing fractions is just as easy as multiplying fractions. In fact, almost all the steps are the same. But there is one extra step that we need to discuss before we can divide. That is the “**reciprocal**”.

The reciprocal switches the values of the numerator and denominator. Many people think of it as “flipping” the fraction upside-down. But remember it has to be a fraction (proper or improper) and not a mixed number. Let’s see how that works.

EXAMPLE 7

Find the reciprocal of $\frac{5}{9}$.

SOLUTION TO EXAMPLE 7

1.	Switch the numbers in the numerator and denominator.	$\frac{9}{5}$

Don’t worry about simplifying right now. We will eventually have to, but not until the end of the entire problem.

Why did we have to know about the reciprocal? Because dividing fractions is the same as “multiplying the first fraction by its reciprocal”. Now that we make it a multiplication problem, we use the same step we used for Examples 5 and 6. So let’s see a few problems.

EXAMPLE 8

Divide $\frac{3}{4} \div \frac{2}{6}$.

SOLUTION TO EXAMPLE 8

1.	Multiply by the reciprocal. **<u>NOTE</u>**: We do not take the reciprocal of the first fraction. The first fraction stays the same. When dividing, we take the reciprocal of the second fraction only!	$\frac{3}{4} \times \frac{6}{2}$
2.	Multiply the numerators	$3 \times 6 = 18$
3.	Multiply the denominators.	$4 \times 2 = 8$
4.	Simplify if you can.	$\frac{18}{8} = \frac{9}{4} = 2\frac{1}{4}$

EXAMPLE 9

Divide $2\frac{1}{3} \div 4$.

SOLUTION TO EXAMPLE 9

1.	Make sure numbers are written as proper or improper fractions. You can change any whole number into an improper fraction by placing a 1 in the denominator.	$\frac{7}{3} \div \frac{4}{1}$
2.	Multiply by the reciprocal.	$\frac{7}{3} \times \frac{1}{4}$
3.	Multiply numerators together.	$7 \times 1 = 7$
4.	Multiply denominators together.	$3 \times 4 = 12$
5.	Simplify the fraction if you can. In this case, we can't so we don't do anything.	$\frac{7}{12}$

TRY THESE – Multiplying and Dividing of Fractions

Perform the given operation. Be sure your answer is in lowest terms.

1. $\frac{3}{4} \times \frac{8}{9}$

2. $\frac{3}{10} \div \frac{5}{8}$

3. $5\frac{5}{7} \times 4\frac{2}{3}$

4. $3\frac{4}{5} \div 2\frac{7}{9}$

5. Chris bought 15 yds of fabric from which to cut sashes for the dance team. If each sash is $\frac{3}{4}$ yds longs, how many sashes can she cut?

Adding Fractions

If you recall from your math classes from way back when, you'll remember that you can only add (or subtract) two fractions if they have the same denominator. Most likely, the problems you'll encounter on the competency exam will require you to find a common denominator so you can perform the operation.

There are many ways to find a common denominator. I am going to show you the easiest one to learn, but this method may also require a little more work at the end in terms of simplifying your final answer.

The easiest way to find the common denominator is to simply multiply both denominators together. But remember if you multiply the denominator by a number, you have to multiply the numerator by the same number to keep it as an equivalent fraction. (See the previous section regarding equivalent fractions on page 13 if you forgot how to do this already.) Let's see a couple of examples finding common denominators.

EXAMPLE 10

Re-write the problem $\frac{2}{3}+\frac{1}{5}$ using common denominators. We do not need to perform the operation at this time.

SOLUTION TO EXAMPLE 10

1.	Multiply the two denominators together. This will be my common denominator.	$3 \times 5 = 15$
2.	Now we have to find the equivalent fraction for $\frac{2}{3}$ but with 15 as the denominator. In this case multiply the numerator and denominator by 5.	$\frac{2 \times 5}{3 \times 5} = \frac{10}{15}$
3.	Now we have to find the equivalent fraction for $\frac{1}{5}$ but with 15 as the denominator. In this case multiply the numerator and denominator by 3.	$\frac{1 \times 3}{5 \times 3} = \frac{3}{15}$
4.	When we re-write the problem, we get	$\frac{10}{15} + \frac{3}{15}$

At this point we are done. But we will talk about how to add fractions after we get those common denominators! So just hang on for a minute.

EXAMPLE 11

Re-write the problem $\frac{3}{4} - \frac{2}{7}$ using common denominators. We do not need to perform the operation at this time. (I know this section is on adding fractions, but I thought I would put in a subtraction problem here since finding common denominators for subtraction is the same concept.)

SOLUTION TO EXAMPLE 11

1.	Multiply the two denominators together. This will be my common denominator.	$4 \times 7 = 28$
2.	Now we have to find the equivalent fraction for $\frac{3}{4}$ but with 28 as the denominator. In this case multiply the numerator and denominator by 7.	$\frac{3 \times 7}{4 \times 7} = \frac{21}{28}$
3.	Now we have to find the equivalent fraction for $\frac{2}{7}$ but with 28 as the denominator. In this case multiply the numerator and denominator by 4.	$\frac{2 \times 4}{7 \times 4} = \frac{8}{28}$
4.	When we re-write the problem, we get	$\frac{21}{28} + \frac{8}{28}$

Okay. Let's move on to adding fractions with common denominators. This is the easy part. We've already done the hard work. To add fractions with common denominators, add the two numerators together. Then place it over the common denominator. Simplify your fraction if necessary, especially if you need to convert it to a mixed number!

Now let's finish the problems we started.

EXAMPLE 10 (revisited)

Add $\frac{2}{3} + \frac{1}{5}$.

SOLUTION TO EXAMPLE 10 (revisited)

We don't need to show all the work of finding the common denominator since we've already done it on the previous page.

1.	When we re-write the problem, we get	$\frac{10}{15} + \frac{3}{15}$
2.	Add the numerators.	$10 + 3 = 13$
3.	Place over the common denominator.	$\frac{13}{15}$
4.	Simplify. In this case, it is in lowest terms already.	

EXAMPLE 12

Add $2\frac{3}{5} + 4\frac{2}{9}$.

SOLUTION TO EXAMPLE 12

1.	Multiply the two denominators together. This will be my common denominator.	$5 \times 9 = 45$
2.	Now we have to find the equivalent fraction for $\frac{3}{5}$ but with 45 as the denominator. In this case multiply the numerator and denominator by 9.	$\frac{3 \times 9}{5 \times 9} = \frac{27}{45}$
3.	Now we have to find the equivalent fraction for $\frac{2}{9}$ but with 45 as the denominator. In this case multiply the numerator and denominator by 5.	$\frac{2 \times 5}{9 \times 5} = \frac{10}{45}$
4.	When we re-write the problem, we get	$2\frac{27}{45} + 4\frac{10}{45}$
5.	Add numerators.	$27 + 10 = 37$
6.	Place over the common denominator and simplify.	$\frac{37}{45}$
7.	Add the whole numbers.	$2 + 4 = 6$
8.	Put it all together.	$6\frac{37}{45}$

EXAMPLE 13

Add $5\frac{1}{4} + 19\frac{11}{12}$.

SOLUTION TO EXAMPLE 13

1.	Multiply the two denominators together. This will be my common denominator.	$4 \times 12 = 48$
2.	Now we have to find the equivalent fraction for $\frac{1}{4}$ but with 48 as the denominator. In this case multiply the numerator and denominator by 12.	$\frac{1 \times 12}{4 \times 12} = \frac{12}{48}$
3.	Now we have to find the equivalent fraction for $\frac{11}{12}$ but with 48 as the denominator. In this case multiply the numerator and denominator by 4.	$\frac{11 \times 4}{12 \times 4} = \frac{44}{48}$
4.	When we re-write the problem, we get	$5\frac{12}{48} + 19\frac{44}{48}$
5.	Add numerators.	$12 + 44 = 56$
6.	Place over the common denominator and simplify.	$\frac{56}{48} = \frac{7}{6} = 1\frac{1}{6}$
7.	Add the whole numbers. Notice we added the 1 whole number from Step #6 above since it was a mixed number.	$5 + 19 + 1 = 25$
8.	Put it all together.	$25\frac{1}{6}$

Subtracting Fractions

Many of the steps for subtracting fractions are similar to those on adding fractions.
In general, the steps for subtracting fractions would be:

1. Find common denominators.
2. Re-write each fraction using the common denominators.
3. Subtract the numerators and place of the common denominator.
4. And simplify if we can.

So let’s take a look at a problem and review our step, but this time we are going to subtract fractions.

EXAMPLE 14

Subtract $5\frac{3}{4} - 1\frac{5}{12}$.

SOLUTION TO EXAMPLE 14

1.	Multiply the two denominators together. This will be my common denominator.	$4 \times 12 = 48$
2.	Now we have to find the equivalent fraction for $\frac{3}{4}$ but with 48 as the denominator. In this case multiply the numerator and denominator by 12.	$\frac{3 \times 12}{4 \times 12} = \frac{36}{48}$
3.	Now we have to find the equivalent fraction for $\frac{5}{12}$ but with 48 as the denominator. In this case multiply the numerator and denominator by 4.	$\frac{5 \times 4}{12 \times 4} = \frac{20}{48}$
4.	When we re-write the problem, we get	$5\frac{36}{48} - 1\frac{20}{48}$
5.	Subtract numerators.	$36 - 20 = 16$
6.	Place over the common denominator and simplify.	$\frac{16}{48} = \frac{1}{3}$
7.	Subtract the whole numbers.	$5 - 1 = 4$
8.	Put it all together.	$4\frac{1}{3}$

Great! Now for the catch! I know. There’s always a catch. What happens when the first fraction is smaller than the second fraction? What do we do? Panic? Of course not. Let’s take a look at a problem and work out the steps.

EXAMPLE 15

Subtract $8\frac{1}{3} - 6\frac{4}{5}$.

SOLUTION TO EXAMPLE 13

1.	Multiply the two denominators together. This will be my common denominator.	$3 \times 5 = 15$
2.	Now we have to find the equivalent fraction for $\frac{1}{3}$ but with 15 as the denominator. In this case multiply the numerator and denominator by 5.	$\frac{1 \times 5}{3 \times 5} = \frac{5}{15}$
3.	Now we have to find the equivalent fraction for $\frac{4}{5}$ but with 15 as the denominator. In this case multiply the numerator and denominator by 3.	$\frac{4 \times 3}{5 \times 3} = \frac{12}{15}$
4.	When we re-write the problem, we get	$8\frac{5}{15} - 6\frac{12}{15}$
5.	Subtract numerators.	$5 - 12 = -7$
6.	Place over the common denominator and simplify.	$\frac{-7}{15}$
7.	Subtract the whole numbers.	$8 - 6 = 2$
8.	Put it all together.	$2 - \frac{7}{15}$

At this point, I am going to stop the problem and talk about "regrouping". Regrouping is a method of changing a mixed number (or whole number) into a fraction with a whole number PLUS an improper fraction. We only use this method to simplify subtraction problems.

In order to regroup a fraction, we also start by saying it is "one less than our number" + 1. What? Don't worry it sounds worse than it actually is.

9.	Regroup the whole number. The first number is going to be our whole number. The "+ 1" we are going to re-write it as "+ fraction with the common denominator". Recall that 1 is just the number over itself. In our case, the number is 15.	$2 = 1 + 1$ $2 = 1 + \frac{15}{15}$
10.	Re-write our problem with the regrouping would look like this.	$1 + \frac{15}{15} - \frac{7}{15}$
11.	Subtract the fractions and simplify again if necessary.	$1 + \left(\frac{15}{15} - \frac{7}{15}\right)$ $1 + \left(\frac{8}{15}\right)$
12.	Put it all together.	$1\frac{8}{15}$

TRY THESE – Adding and Subtracting Fractions

Perform the given operation. Be sure your answer is in lowest terms.

1. $\frac{3}{4}+\frac{8}{9}$

2. $7\frac{3}{10}+12\frac{5}{8}$

3. $\frac{5}{7}-\frac{2}{3}$

4. $8\frac{3}{5}-2\frac{1}{4}$

5. $5\frac{7}{10}-4\frac{5}{6}$

6. $13\frac{2}{5}-4\frac{7}{9}$

7. Joe has a piece of board that is 8 ft. long. He cuts off a piece of the board that is $4\frac{5}{16}$ ft. in length. How much of the board does Joe have left?

CHAPTER 5: EXPONENTS

There are seven basic rules of exponents. If you understand how to apply each of these rules, then you'll have no problem handling the problems that involve exponents on the Math Competency Exam. The names of these rules are the way I remember them. They are not always named this way in textbooks. So you might not remember (or never taught) using these names.

Product Rule: $x^m x^n = x^{(m+n)}$	$x^5 x^2 = x^{(5+2)} = x^7$
Quotient Rule: $\frac{x^m}{x^n} = x^{(m-n)}$	$\frac{x^{10}}{x^4} = x^{(10-4)} = x^6$
Power Rule: $(x^m)^n = x^{(mn)}$	$(x^5)^3 = x^{(5\cdot 3)} = x^{15}$
Power of Product Rule: $(xy)^k = x^k y^k$	$(xy)^5 = x^5 y^5$
Power of Quotient Rule: $\left(\frac{x}{y}\right)^m = \frac{x^m}{y^m}$	$\left(\frac{x}{y}\right)^4 = \frac{x^4}{y^4}$
Zero Exponent: $x^0 = 1 \; where \; x \neq 0$	$5^0 = 1$
Negative Exponent: $x^{-k} = \frac{1}{x^k}$	$x^{-2} = \frac{1}{x^2}$

NOTE: The Product Rule and Quotient Rules require that you have the same "base". In both examples, "x" is the base. When working out more difficult problems involving exponents, break the problem down to simpler problems. Then apply the rules of exponents.

EXAMPLE 1	$(2y^3)(3y^7)$	$= (2)(3)(y^{3+7})$	$6y^{10}$
EXAMPLE 2	$(-3ab^2c^5)(-5a^2b^3c)$	$= (-3)(-5)(a^{1+2})(b^{2+3})(c^{5+1})$	$15a^3b^5c^6$
EXAMPLE 3	$\frac{10x^3y^6}{4x^2y}$	$= \frac{10}{4} \cdot \frac{x^3}{x^2} \cdot \frac{y^6}{y} = \frac{5}{2} \cdot (x^{3-2})(y^{6-1})$	$\frac{5xy^5}{2}$
EXAMPLE 4	$(2^3)^2$	$= 2^6 = 2 \times 2 \times 2 \times 2 \times 2 \times 2$	64

EXAMPLE 5	$\left(\frac{2x^2}{y^4}\right)^2$	$=\frac{2^2(x^2)^2}{(y^4)^2}$	$\frac{4x^4}{y^8}$
EXAMPLE 6	$\frac{12xy^6}{4x^0y^6}$	$=\frac{12}{4}\cdot\frac{x}{x^0}\cdot\frac{y^6}{y^6}=3\cdot x^{(1-0)}\cdot y^{(6-6)}$	$3x$
EXAMPLE 7	$(-3a^3b)^0(4ab^4)$	$=1\cdot(4ab^4)$	$(4ab^4)$
EXAMPLE 8	$4x^{-4}y^2$	$=4\cdot\frac{1}{x^4}\cdot y^2$	$\frac{4y^2}{x^4}$

TRY THESE – Properties of Exponents

1. $(3xy^5z^2)(2x^0y^{-3}z)$

2. $\frac{30a^5b^2}{24a^4b^2}$

3. $\left(\frac{4x^5}{y^2}\right)^2$

4. $(2x^3y^5z)^3$

5. $\frac{4a^4b^7}{6a^5b^6}$

6. $(2q^4r^2s^7)(-2q^{-3}r^4s^3)$

7. $(-3x^3)^2(2x^{-1})$

8. $\left(\frac{3x^{11}y^4}{5z^2}\right)^2$

9. $\frac{2x^3y^2z^9}{6x^3y^3}$

CHAPTER 6: SOLVING EQUATIONS

Solving equations is a skill that is tested many times on WSU's Math Competency Exam. Additionally, many of the problems that ask you to solve an equation will invariably involve many of the other skills discussed in the other chapters in this study guide. You will also find the solving equations will be found in all math courses if it is MAT 0993 or MAT 7000. Knowing how to break down an equation and solve for a variable is inevitable to succeed.

One important idea to remember about equations is that two expressions are equal. I always imagine a seesaw (or some people call it a teeter-totter). If the two people on either side are the same weight, the seesaw is balanced (or equal).

But what happens when one side is heavier than the other? You guessed it, one side goes up and the other side goes down. The seesaw is now out of balance and no longer equal. To get it back to being equal, we need to either remove something from the heavier side or add more to the lighter side.

That is what an equation is all about…keeping both sides equal. So how do we do this in mathematics? Good question. Let's see.

When solving an equation we have one ultimate goal in mind. That goal is to "Isolate the variable". The variable is the "unknown" and could be represented by a number of different letters like x, y, z or a to name a few. Let us look at a rather simple example.

EXAMPLE 1

Solve: $x + 3 = 7$

I have included a picture to visualize the equation. Notice that the balance is equal. What would happen if I took one of the red marbles from the left side? The right side would be heavier and drop down. To balance it out, I need to take one red marble from the right. This brings us to a couple of rules of equations.

RULE 1: You may add (or subtract) the same number from both sides of the equal sign.

RULE 2: You may multiply (or divide) the same number from both sides of the equal sign.

The overall rule I like to remember is "***What you do to one side of the equal sign, you must do the same thing to the other side***".

SOLUTION TO EXAMPLE 1

Remember our goal is to get the variable, x, by alone.

1.	$x + 3 = 7$	Start with our equation. The variable is x. To get x by itself, we have to get rid of the "+ 3".
2.	$x + 3 - 3 = 7 - 3$	To get rid of the "+ 3" we subtract 3 from both sides.
3.	$x = 4$	Simplify.

When we say "get rid of" something we don't mean simply delete it from the problem. We must do the opposite operation to whatever we are trying to "get rid of", but we do that operation to BOTH sides of the equal sign to ensure things stay balanced. Combining numbers is not the same as "getting rid of" them. In the example above we combine "7 – 3" to equal "4". We combined the two numbers together.

Let's take a look at another example.

EXAMPLE 2

Solve: $2x + 4 = 10$

SOLUTION TO EXAMPLE 2

Remember our goal is to get the variable, x, by alone.

1.	$2x + 4 = 10$	
2.	$2x + 4 - 4 = 10 - 4$ $2x = 6$	To get rid of the "+ 4" we subtract 4 from both sides.
3.	$\frac{2x}{2} = \frac{6}{2}$	To get rid of the "2 times", we need to divide both sides by 2.
4.	$x = 3$	Simplify.

The beautiful thing about solving equations is that you can always check to see if you came up with the correct answer. To check if "3" is the correct answer, simply "plug" 3 back into the original equation for x and see if it makes sense.

5.	$2(3) + 4 \stackrel{?}{=} 10$	
6.	$6 + 4 \stackrel{?}{=} 10$	
7.	$10 = 10$	Since both sides are equal, $x = 3$ is the correct answer.

As you may think, equations they'll ask you to solve on the math competency exam aren't quite as simple, but the general method we used above works on virtually all of the problems you'll encounter. They will likely be a little more complex.

Let's look at a problem taken directly from the practice exam.

EXAMPLE 3
Solve: $3(2-y)-4=4-5(y+1)$

SOLUTION TO EXAMPLE 3
Even though it looks a little more involved than the other examples we've seen, it isn't anything we cannot handle. But where do we begin?

1.		$3(2-y)-4=4-5(y+1)$
2.	Distribute the numbers in front of the parentheses.	$3(2)-3(y)-4=4-5(y)+(-5)(1)$ $6-3y-4=4-5y-5$
3.	Combine like terms on each side of the equal sign.	$6-4-3y=4-5-5y$ $2-3y=-1-5y$
4.	Notice that there are variables on both sides of the equal sign. We need to get them both on the same side. It does not matter which side. In this case, I will get rid of the "$-5y$" from both sides.	$2-3y+5y=-1-5y+5y$ $2+2y=-1$
5.	Now we still have to get the "y" variable by itself. Let's get rid of the 2	$2-2+2y=-1-2$ $2y=-3$
6.	Lastly, we need to divide both sides by 2.	$\frac{2y}{2}=\frac{-3}{2}$ $y=\frac{-3}{2}$
7.	Remember to plug the number back into the original equation to check if it is reasonable.	$3\left(2-\frac{-3}{2}\right)-4=4-5\left(\frac{-3}{2}+1\right)$

In high school, most equations that you solved might have a nice whole number as an answer. Most students expect that when solving equations. Notice that the answer for EXAMPLE 3 was a fraction. Just because you get a fraction does not mean you got an incorrect answer. Be confident with your steps and you'll do fine.

Let's take a look at one last type of equation you might be asked to solve on the math competency exam. In EXAMPLE 4, you will be asked to solve for a variable. Don't be afraid of this type of equation. You follow the same procedures we have worked with in other examples. The only difference is we are using mainly variables instead of actual numbers.

EXAMPLE 4

Solve for T: $V = \frac{2P+T}{T}$

SOLUTION TO EXAMPLE 4

Even though it looks a little more involved than the other examples we've seen, it isn't anything we cannot handle. But where do we begin?

1.		$V = \frac{2P+T}{T}$
2.	Remember a fraction is another way to write a division problem. So I am going to switch this to a division before moving on.	$V = (2P+T) \div T$
3.	Now we can get rid of the "$\div T$" by multiplying both sides by T.	$V \cdot T = (2P+T) \div T \cdot T$ $V \cdot T = 2P + T$
4.	The next step would be to get the T's on one side of the equal sign. So I am going to subtract the T from both sides. We cannot combine the "$V \cdot T - T$" since there is a multiplication and an addition sign. So be careful here!	$V \cdot T - T = 2P + T - T$ $V \cdot T - T = 2P$
5.	What we can do though is to use the distributive property. This allows us to "divide" or "factor out" the T from the expression.	$T(V-1) = 2P$
6.	We have one last operation to do. To get the T by itself, we need to divide the parentheses from both sides.	$\frac{T(V-1)}{(V-1)} = \frac{2P}{(V-1)}$
7.	So our final answer will be this equation.	$T = \frac{2P}{(V-1)}$

In the other examples, we told you to plug the answer back into the original equation to see if it makes sense. But in this case, we don't just have one numeric value we can plug into the problem. We have this expression. If you are ambitious, you can plug it in and see if it works out. But I would suggest going over step-by-step to make sure your steps are algebraically sound.

TRY-THESE – Solving Equations

1. $4(2x+3)=7x+1$

2. $3x-12=-12+6x$

3. $\frac{1}{3}x-\frac{1}{4}=\frac{5}{12}+\frac{1}{2}x$

4. $3(6x-2)+2=2(2x-4)$

5. $\frac{2}{3}(6x-2)=5$

6. Solve for R: $2L-4R+2=4L-2R$

7. Solve for Q: $\frac{Q-4}{4}=2Q-4R$

8. $3x-(4x-3)+2=2(4x-2)$

9. $Solve\ for\ V:\quad -2(V-3)+4V=2R$

10. $2x-3+\frac{1}{2}x=2x+6$

CHAPTER 7: INEQUALITIES

You will be asked to solve problems that involve inequalities. Those are the problems that use the following symbols. I have also included their meanings.

<	"Less than"
≤	"less than or equal to"
>	"greater than"
≥	"greater than or equal to"

Lucky for us, the way to solve inequalities is exactly the same way we solve equations. The only difference is that an inequality uses one of the symbols above. We still try to isolate the variable on one side of the inequality symbol. But there is one rule that we need to remember with inequalities. The key rule when solving inequalities is "if you multiply OR divide both sides of the inequality by a negative number, you MUST reverse the inequality sign." You have to do this reversal every time you multiply or divide by a negative number. Most problems you would only have to do this once. So just be careful when you are solving inequalities. Don't make a mistake by reversing the inequality when you add or subtract. This is a common mistake made by a lot of students.

EXAMPLE 1

Solve $3 > \frac{4+u}{-5}$

SOLUTION TO EXAMPLE 1

1.	Multiply both sides by (-5). Since we are multiplying by a negative value, remember to reverse the inequality sign.	$3 \cdot (-5) < \frac{4+u}{-5} \cdot (-5)$ $-15 < 4 + u$
2.	Subtract 4 from both sides. Since we subtracted, we do not need to reverse the inequality.	$-15 - 4 < 4 - 4 + u$ $-19 < u$
3.	Some professors are fine by the answer as it is. But mathematically, we want to have the variable on the left side. So we need to make a little adjustment. Since our answer has the inequality opening up to the "u", when we switched it around, the inequality is still opening up to the "u".	$u > -19$

Now that we have an answer, what does the inequality actually mean? Our answer for EXAMPLE 1 is $u > -19$. That means that I can pick any number greater than -19 and when I plug it back into the inequality, it will make sense. Try a couple of numbers and see.

Again, solving inequality problems are really no different than solving equations. You simply have to remember the multiplying or dividing by a negative number rule.

TRY THESE – Solving Inequalities

1. $$4(3x-5)+2 \geq 13x+30$$

2. $$3x-5<6x+25$$

3. $$11(2x-1) \leq 33$$

4. $$3x+3>\frac{4x-3}{-2}$$

5. $$2x-4(2x+2)>2x+12$$

6. $$\frac{3x-5}{2} \leq \frac{7x}{4}$$

CHAPTER 8: RATIOS

A **ratio** is simply a comparison of two numbers. Ratios can be written in three different ways. Let's look at a quick example and show you the different ways of writing ratios.

EXAMPLE 1

There are 10 boy and 12 girls in a math class. Write the ratio of boys to girls in three different ways.

SOLUTION TO EXAMPLE 1

First, since boys were listed first in the ratio, the number of boys will be listed first. If the girls were listed first, their number would have been listed first. These are the labels that make sense to me and you probably won't find them in any books.

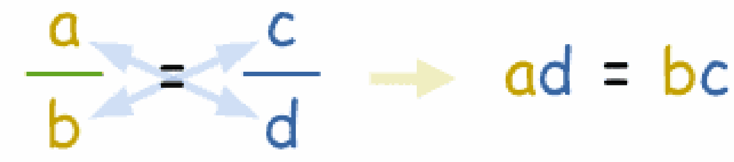
1.	Fraction	$\frac{10\ boys}{12\ girls}$
2.	Colon	$10\ boys: 12\ girls$
3.	Words	$10\ boys\ to\ 12\ girls$

When it comes to working with ratios, fractions is the easiest method to use. This being said, when you write a ratio as a fraction, you can use some of the properties of fractions with the ratio. For example, we can simplify the ratio $\frac{10\ boys}{12\ girls}$ as if it were a fraction. Divide the numerator and denominator by 2. $\frac{10\div 2}{12\div 2}=\frac{5}{6}$. These are **equivalent ratios**.

Solving Proportions

A **proportion** is an equation that states that two ratios are equivalent.

To solve proportions, we start by putting two ratios equal to each other. Then we use a technique called cross-multiplication to re-write the equation.



Let's take a look at an example and see how to apply the cross-multiplication.

EXAMPLE 2

Solve the proportion. $\frac{n}{16} = \frac{15}{24}$

SOLUTION TO EXAMPLE 2

1.	Apply the cross-multiplication.	$n \cdot 24 = 24n$ $16 \cdot 15 = 240$
2.	Write the equation and solve.	$24n = 240$ $\frac{24n}{24} = \frac{240}{24}$ $n = 10$

Here's an example from the math competency exam.

EXAMPLE 3

To obtain a certain shade of pink, a painter must mix 7 parts red paint with 3 parts white paint. If the painter uses 28 gallons of red paint, how many gallons of white paint are needed?

SOLUTION TO EXAMPLE 3

1.	Create our proportion. When writing your proportion make sure you put the same items in the numerator and the same items in the denominator. In this problem, I put the red pain in the numerator.	$\frac{7\ red}{3\ white} = \frac{28\ red}{x\ white}$
1.	Apply the cross-multiplication.	$7 \cdot x = 7x$ $3 \cdot 28 = 84$
2.	Write the equation and solve.	$7x = 84$ $\frac{7x}{7} = \frac{84}{7}$ $x = 12$

∴The painter will need 12 cans of white paint.

TRY-THESE – Proportions

1. $\frac{h}{15} = \frac{20}{75}$

2. $\frac{13}{10} = \frac{52}{x}$

3. $\frac{78}{m} = \frac{18}{12}$

4. $\frac{g}{39} = \frac{4}{12}$

5. $\frac{2.4}{32} = \frac{y}{16}$

6. $\frac{4.5}{6} = \frac{a}{20}$

7 A marathoner can run 1 mile in 7.56 minutes. How long will it take him to complete a marathon (26.2 miles)?

CHAPTER 9: PERCENTAGES

There aren't too many problems on the math competency exam that deal with percentages. You can count on one for sure. We won't spend too much time dealing with percentages. However, the problem(s) they do present you with shouldn't be too bad. If you can solve an equation, you can deal with the percentage problems presented on the math competency exam.

The word **percent** is derived from the Latin "per centum" which translates to "by the 100". When we talk about percent, we can think about them as a fraction out of 100.

45% means $\frac{45}{100}$ which can then be turned into a decimal, 0.45.
6% means $\frac{6}{100}$ which can then be turned into a decimal, 0.06.

There are three types of percent problems. We will take a look at each individually.

1. Finding the percent of a number
2. Finding what percent one number is of another
3. Finding a number when a percent of it is known

When we translate English statements into math expressions, we learned that

- "of" refers to "multiplication"
- "is" refers to the equal sign

Finding the Percent of a Number

To find the percent of a number, we should write and solve an equation. We do this by changing the English statement into math expressions.

EXAMPLE 1
What number is 60% of 45?

SOLUTION TO EXAMPLE 1

1.	"What number" is the unknown number we are trying to find. We replace this phrase by a variable. "is" refers to the equal sign. Change the percentage to its fraction. So 60% is $\frac{60}{100}$ "of" refers to multiplication.	$x = \frac{60}{100} \times 45$
2.	Solve the equation.	$= \frac{60}{100} \times \frac{45}{1} = \frac{2700}{100} = 27$

Let's take a look at one of the problems on the math competency exam practice test.

EXAMPLE 2
Marcy wants to buy a car for $3600. A down payment of 35% is required. How much is the down payment?

SOLUTION 1 TO EXAMPLE 2
When we read the problem, it says we have to put a down payment which is a percentage of the cost of the car. So it is really asking *"How much is 35% of $3600?"*

1.	"How much" is the unknown number we are trying to find. We replace this phrase by a variable. "is" refers to the equal sign. Change the percentage to its fraction. So 35% is $\frac{35}{100}$ "of" refers to multiplication.	$x = \frac{35}{100} \times 3600$
2.	Solve the equation.	$= \frac{35}{100} \times \frac{3600}{1} = \frac{126000}{100} = \$1260$

SOLUTION 2 TO EXAMPLE 2
This solution works best when the percent ends with a "0" or "5", in this case "35%". What I know already is that 10% of $3600 is $360. I had to move the decimal one space to the left. 5% would be half of 10%. So 5% of $3600 is $180.

1.	What I know already is that 10% of $3600 is $360. I had to move the decimal one space to the left. 5% would be half of 10%. So 5% of $3600 is $180.	What number is 10% of $3600? What number is 5% of $3600?
2.	To get 35%, we need to find 10% + 10% + 10% + 5%.	$360 + $360 + $360 + $180 = $1260

Finding What Percent One Number is of Another

We still use the same translations that we did for "Finding the Percent of a Number". But our equation is going to be a little different.

Let's jump right into our examples and I'll point out the differences.

EXAMPLE 3

What percent of 65 is 13?

SOLUTION TO EXAMPLE 3

1.	"what percent" is the unknown number we are trying to find. We replace this phrase by a variable. "of" refers to multiplication. "is" refers to the equal sign.	$p \times 65 = 13$
2.	Solve the equation.	$p \times 65 \div 65 = 13 \div 65$ $p = \frac{13}{65} = \frac{1}{5} = 0.20$
3.	Change the decimal to a percent.	0.20 = 20%

∴ So 20% of 65 is 13.

Here is an example from the math competency exam.

EXAMPLE 4

28 is what percent of 70?

SOLUTION TO EXAMPLE 4

1.	"is" refers to the equal sign. "what percent" is the unknown number we are trying to find. We replace this phrase by a variable. "of" refers to multiplication.	$28 = p \times 70$
2.	Solve the equation.	$28 \div 70 = p \times 70 \div 70$ $p = \frac{28}{70} = \frac{2}{5} = 0.40$
3.	Change the decimal to a percent.	0.40 = 40%

∴ So 28 is 40% of 70.

Notice that the two problems where written a little differently, but each problem asks for the same thing, "what percent".

Finding a number when a percent of it is known

We still use the same translations that we did for “Finding the Percent of a Number”. There isn’t much difference except for the equations that we write. You would need to read the question careful and decide which part we are trying to find.

Let’s jump right into our examples.

EXAMPLE 5

4.2% of what number is 2.31?

SOLUTION TO EXAMPLE 5

1.	Change percent to a fraction. “of” refers to multiplication. “what percent” is the unknown number we are trying to find. We replace this phrase by a variable. “is” refers to the equal sign.	$\frac{4.2}{100} \times n = 2.31$
2.	Solve the equation. Remember we are getting rid of a fraction, we multiply by its reciprocal	$\frac{100}{4.2} \times \frac{4.2}{100} \times n = \frac{100}{4.2} \times 2.31$ $n = \frac{100}{4.2} \times \frac{2.31}{1} = \frac{231}{4.2}$
3.	Simplify	$\frac{231}{4.2} = 55$

∴ So 4.2% of 55 is 2.31.

EXAMPLE 6

Joanna took Len to a restaurant for lunch. She left a tip of $5.25, 15% of the bill. How much was the bill for lunch?

SOLUTION TO EXAMPLE 6

When reading over the problem, it is simply asking “$5.25 is 15% of what number?”.

1.	“is” refers to the equal sign. Change percent to a fraction. “of” refers to multiplication. “what percent” is the unknown number we are trying to find. We replace this phrase by a variable.	$5.25 = \frac{15}{100} \times n$
2.	Solve the equation. Remember we are getting rid of a fraction, we multiply by its reciprocal	$\frac{100}{15} \times 5.25 = \frac{100}{15} \times \frac{15}{100} \times n$ $\frac{100}{15} \times \frac{5.25}{1} = n$
3.	Simplify	$\frac{525}{15} = 35$

∴ So $5.25 is 15% of $35.

TRY THESE – Percentages

1. 20% of 80 is what number?

2. 12 is 60% of what number?

3. What percent of 20 is 15?

4. 10 is 60% of what number?

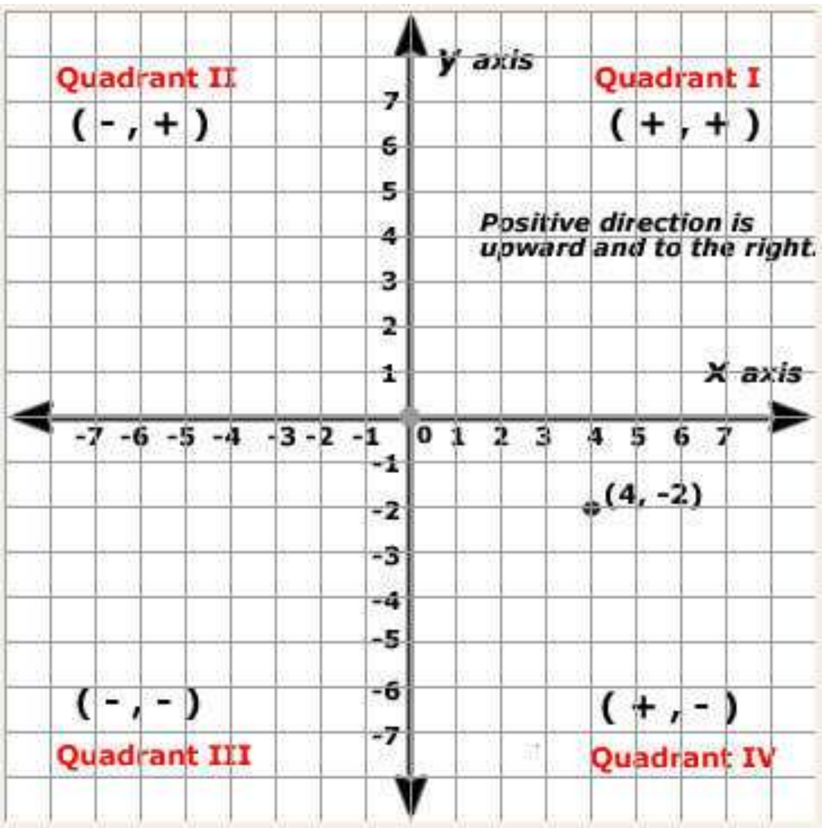
5. What percent of 40 is 8?

6. 70% of the seats at Adams Field were filled with fans. If there were 4200 people who attended the football game, what is the seating capacity of the stadium?

CHAPTER 10: THE COORDINATE PLANE AND ORDERED PAIRS

Coordinate Plane

The **coordinate plane** is based on two perpendicular lines (called **axes**). The horizontal line is called the **x-axis** and the vertical line is called the **y-axis**. The point where the two lines intersect is called the **origin**. The coordinate plane is then divided into four **quadrants**.



You can find any point on the coordinate plane by finding two values, the x-coordinate and the y-coordinate. The **x-coordinate** is how many spaces the point is to the left or right of the origin. The **y-coordinate** is how many spaces the point is above or below the origin. If you look on the coordinate plane above, the point (4, -2) is found 4 spaces to the right of the origin (x-coordinate is 4) and 2 spaces below the origin (y-coordinate is -2).

Remember that in an **ordered pair**, the x-coordinate comes first and the y-coordinate comes second.

(x-coordinate, y-coordinate)

Many people get the coordinates confused. Just remember left/right is first and the x-axis is left/right. Up/down is second and the y-axis is up/down.

CHAPTER 11: GRAPHING LINEAR EQUATIONS

You will be asked to graph several problems on the math competency exam. But remember that it is all multiple choice. So you will have to be able to pick the best distractor. So here are a few pieces of information you may need when working with graphs.

Slope

Slope is the steepness of a line, which just means how fast does it move up or move down. It is a ratio of its rise over its run. It can also be described as the change in the y-direction over the change in the x-direction.

$$\text{slope} = \frac{rise}{run} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{\Delta \text{y}}{\Delta x}$$

That's a lot of symbols to know, but they all mean the same thing.

A positive slope means that the line is increasing as we move from left to right on the coordinate plane.
A negative slope means that the line is decreasing (going down) as we move from left to right on the coordinate plane.
A zero slope is a horizontal line and is given by the equation $y =$"some number"
An undefined slope is a vertical line and is given by the equation $x =$"some number".

The equations that involve perfectly horizontal and vertical lines are quite simple. There is not much to do. It would be helpful to memorize what these types of lines generally look like and if it is horizontal or vertical.

I thing that you will need to find is the slope of a line if you are given only two points. To do this we use the formula $\text{slope} = \frac{y_2 - y_1}{x_2 - x_1}$. Because we have to points, we can't simply say y-coordinate or the x-coordinate. You wouldn't know from which point to take the value. That's what those little 1's and 2's are. y_2 mean the y-coordinate from the 2nd point. So y_1 must mean the y-coordinate from the 1st point. Similarly x_2 mean the x-coordinate from the 2nd point. And x_1 means the x-coordinate from the 1st point. It does not matter which point you select as the 1st point and 2nd point. You just cannot change your mind in the middle of the problem.

EXAMPLE 1

Find the slope between the points (3, 5) and (6, 3).

SOLUTION TO EXAMPLE 1

1.	I usually place the variables above the points so I know exactly which variable goes with the number.	$\begin{matrix}(x_1,y_1)\\(3,5)\end{matrix}$ $\quad$ $\begin{matrix}(x_2,y_2)\\(6,3)\end{matrix}$
2.	Plug the numbers into the formula.	$\frac{3-5}{6-3}$
3.	Simplify the expression. Keep it as a fraction since slope is often written as the ratio.	$\frac{-2}{3}$

It might be helpful to use parentheses when plugging in your numbers. It might seem silly if you have all positive number in your two ordered pairs, but once you start getting negative numbers in the pairs, it is extremely easy to screw up.

If you are working a problem and get a zero in the denominator, the slope is undefined. If you get a number other than zero in the denominator and get a zero in the numerator, the slope is zero.

Slope-Intercept Form

Slope-intercept form is one way that we can write a linear equation. Slope-intercept form is given by the equation $y = mx + b$. Notice the "y" variable is by itself. That means in problems we might have to solve for "y" if it is not by itself.

Why are we making a big deal about slope-intercept form? It is because slope-intercept form gives us two important pieces of the graph of the linear equation. It tells us the slope of the line and also the **y-intercept** (the point where the line crosses over the y-axis).

The slope is represented by the variable "m" and the y-intercept is the point $(0, b)$.

So let's use the slope intercept form.

EXAMPLE 2

Find the slope of the following line: $2x + 4y = 10$

SOLUTION TO EXAMPLE 2

1.	Let's solve for the variable "y". If you need a refresher, look back in Example 4 of Chapter 6.	$2x + 4y = 10$ $2x - 2x + 4y = -2x + 10$ $4y = -2x + 10$ $\frac{4y}{4} = \frac{-2x}{4} + \frac{10}{4}$ $y = \frac{-1}{2}x + \frac{5}{2}$
2.	Now that "y" is by itself, we can find our slope and y-intercept. Slope is the number multiplied by x.	$\text{Slope } is -\frac{1}{2}$
3.	The y-coordinate of the y-intercept is "b".	$\text{y-intercept is } \left(0, \frac{5}{2}\right)$

Let's do one more "Slope-intercept Form" problem.

EXAMPLE 3

Find the slope of the following line: $3x + 7y = 14$

SOLUTION TO EXAMPLE 3

1.	Let's solve for the variable "y". If you need a refresher, look back in Example 4 of Chapter 6.	$3x + 7y = 14$ $3x - 3x + 7y = -3x + 14$ $7y = -3x + 14$ $\frac{7y}{7} = \frac{-3x}{7} + \frac{14}{7}$ $y = \frac{-3}{7}x + 2$
2.	Now that "y" is by itself, we can find our slope and y-intercept. Slope is the number multiplied by x.	$\text{Slope } is -\frac{3}{7}$
3.	The y-coordinate of the y-intercept is "b".	$\text{y-intercept is } (0,2)$

TRY THESE – Finding Slope and y-Intercept

1. Find the slope between the points $(-3,4)$ and $(-2,-5)$

2. Find the slope between the points $(4,6)$ and $(4,-2)$

3. Find the slope between the points $(5,2)$ and $(-1,2)$

4. Find the slope between the points $(5,9)$ and $(-5,8)$

5. Find the slope and y-intercept of the line
$$3y = 3x - 9$$

6. Find the slope and y-intercept of the line
$$-4x - 3y = 12$$

7. Find the slope and y-intercept of the line
$$x = 12$$

8. Find the slope and y-intercept of the line
$$3x - y = -4$$

Graphing Linear Equations

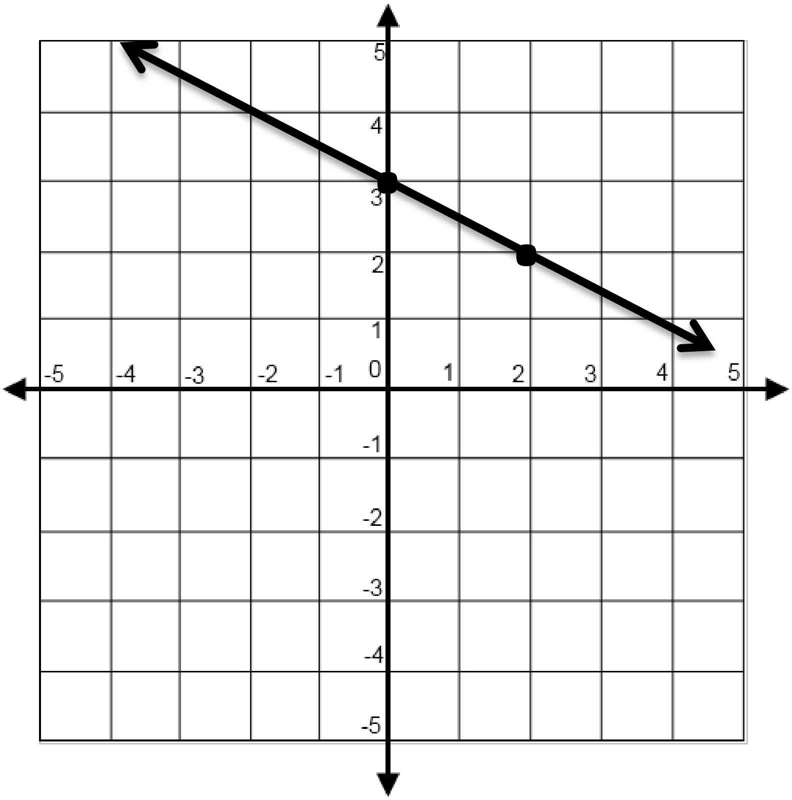
There are many different approaches you can take to graphing a linear equation. In order to graph a line, you have to have two points. Slope-Intercept Form is not always the easiest method to do, but it works almost every single time. So we need to make sure we solve our linear equation for the “y” variable. This will give us the slope of our line and the y-intercept (one of our two points that we need). As we go through the next example, I will walk you through how to get the second point.

EXAMPLE 4

Graph the line $2x + 4y = 12$

SOLUTION TO EXAMPLE 3

1.	Let's solve for the variable "y". If you need a refresher, look back in Example 4 of Chapter 6.	$2x + 4y = 12$ $2x - 2x + 4y = -2x + 12$ $4y = -2x + 12$ $\frac{4y}{4} = \frac{-2x}{4} + \frac{12}{4}$ $y = \frac{-1}{2}x + 3$
2.	Now that "y" is by itself, we can find our slope and y-intercept. Slope is the number multiplied by x.	Slope $is \frac{-1}{2}$
3.	The y-coordinate of the y-intercept is "b".	y-intercept is $(0,3)$
4.	So now that we have the slope and y-intercept, let's move to our graph.	



5.	First plot the y-intercept. Find the origin and count up that many spaces. Put a point there.
6.	Remember that slope is the rise over the run. So the top number tell us how far to move either up or down. Since the number is a negative one, we move one space down.
7.	The run is a positive two so we would move the point two space to the right.
8.	Draw your second point.
9.	Last thing we need to do is connect the points.

TRY THESE – Graphing Linear Equations

Given the linear equation, find the slope and y-intercept. Then graph on a coordinate plane.

1. $y = -3x + 1$

2. $x = 5$

3. $2x - 5y = 10$

4. $y = -2$

5. $y = \frac{1}{2}x - 3$

6. $4x + 4y = 8$

7. $x = -1$

8. $3x - 7y = 7$

CHAPTER 12: SQUARES AND SQUARE ROOTS

The phrase "squared" when dealing with exponents actually does come from the geometric shape. To find the area of a square, you would multiply the base by its height. In a square, the base and the height are the same length. So the term "squared" refers to multiplying a number by itself.

Perfect Squares

For those that don't remember perfect squares, I have included a chart to refresh your memory.

n	Multiplication	Square expression	Perfect square value
1	1×1	1^2	1
2	2×2	2^2	4
3	3×3	3^2	9
4	4×4	4^2	16
5	5×5	5^2	25
6	6×6	6^2	36
7	7×7	7^2	49
8	8×8	8^2	64
9	9×9	9^2	81
10	10×10	10^2	100
11	11×11	11^2	121
12	12×12	12^2	144
13	13×13	13^2	169
14	14×14	14^2	196
15	15×15	15^2	225

So when we multiply a whole number by itself, we get a **perfect square** value.

Square Roots

A square root is the value we get when we work our way backwards from the perfect square value. If we break down the term "square root", the word "square" refers to the perfect square number. The word "root" refers to the number we start with, n. The square root of 4 is 2. In mathematical symbols it will look like this: "$\sqrt{4} = 2$".

For Future Reference: If you think back to multiplication, a positive number times a positive number equals a positive number. A negative number times a negative number equals a positive number. So when we take the square root of a number, we actually get two roots, a positive and a negative.

We know that $\sqrt{36} = 6$ since $6 \times 6 = 36$. But $(-6) \times (-6) = 36$ also. So $\sqrt{36} = -6$ is also a solution. Therefore, $\sqrt{36} = 6\ and - 6$. Usually we only use the positive square root. But your math classes might also ask for the negative square root value.

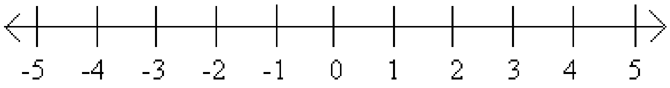
If we use a perfect square value, we get a whole number for our square root. That's great! But what happens when we are not given a perfect square value? Since we don't have a calculator to give us the decimal we want, we have to learn how to estimate a square value.

This is part of a problem from the practice math competency exam.

EXAMPLE 1

Place the following number in the appropriate place on the number line.

$$\sqrt{15}$$



SOLUTION TO EXAMPLE 1

If we look at our table of perfect squares, 15 is not on the table. But we should notice that 15 is found between 9 and 16 (which are perfect squares). That means that $\sqrt{15}$ must be found in between $\sqrt{9}$ and $\sqrt{16}$. With me so far? From our chart, $\sqrt{9} = 3$ and $\sqrt{16} = 4$. So putting our logic all together, $\sqrt{15}$ has to be in between 3 and 4 on the number line.

Since 15 is closer to 16, $\sqrt{15}$ is closer to 4. When we place our point on the number line, we should place it closer to 4.

TRY THESE – Estimating Square Roots

1. $\sqrt{51}$
2. $\sqrt{121}$
3. $\sqrt{49}$
4. $\sqrt{5}$
5. $\sqrt{99}$
6. $\sqrt{34}$
7. $\sqrt{150}$
8. $\sqrt{\frac{16}{25}}$

NOTE: Notice that there is a fraction in Problem 8. Hopefully you didn't freak out at the sight. But there is one helpful property that you would need to know. If you ever have a fraction in the square root, you can to the square root of its numerator and place it over the square root of its denominator.

$$\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$$

Go back and try Problem 8 if you haven't already done so.

CHAPTER 13: POLYNOMIALS

We will not cover all there is to know about polynomials for the math competency exam. We will go over the addition, subtraction, and multiplication of polynomials. We will not go over factoring in this study guide. We will go over it in the study guide for PART B.

So let's begin with a few definitions that will help us understand polynomials.

A **term** (or **monomial**) is an expression that is either a single number, a variable, or the product of a number and one or more variables. So examples of terms are $4, x, 3y, 2y^2, 5xy, 6x^3y^2$.

A **polynomial** is the sum of two or more monomials. Each monomial is called a **term** of the polynomial and are separated by either a "+" or "-" sign.

A polynomial is written in **standard form** when its terms are arranged in order from the greatest or least powers of one of the variables.

Like terms are terms that have exactly the same variable combination (exponents and all). Some examples of like terms are $x^2y, 5x^2y, and -3x^2y$. Notice that each of the terms have an "x" variable with a power of 2 and a "y" variable.

These are NOT like terms: $xy, x^2y, 2xy^2$
Even though they all have an "x" variable and a "y" variable, their exponents aren't always the same.

We really need to pay attention to which terms are like terms when we add or subtract polynomials.

Adding and Subtracting Polynomials

To add polynomials, we combine the like terms together. That's why it's important to know about the like terms.

EXAMPLE 1
Simplify. $(4x^2 + 3x - 2) + (3x^2 + 7)$

SOLUTION TO EXAMPLE 1

1.	Add the like terms together.	$(4x^2 + 3x - 2) + (3x^2 + 7)$ $(4x^2 + 3x^2) + 3x + (-2 + 7)$
2.	Simplify.	$(4x^2 + 3x^2) = 7x^2$ $(-2 + 7) = 5$
3.	Put it all together.	$7x^2 + 3x + 5$

NOTE: When I added the like terms, I only added the coefficients (the numbers in front of the variable). The variable and exponent stayed the same. Remember the property of exponents. You only add the exponents together when you multiply.

Subtraction of polynomials is very similar to adding of real numbers. To subtract one polynomial from another, add its opposite and simplify.

EXAMPLE 2

Simplify. $(5x^2 - 11) - (3x^2 - 3x + 2)$

SOLUTION TO EXAMPLE 2

1.	Subtract the like terms together.	$(5x^2 - 11) - (3x^2 - 3x + 2)$ $(5x^2 - 3x^2) + \big(0x - (-3x)\big) + \big((-11) - 2\big)$
2.	Simplify.	$(5x^2 - 3x^2) = 2x^2$ $\big(0x - (-3x)\big) = 0x + 3x = 3x$ $\big((-11) - 2\big) = -13$
3.	Put it all together.	$2x^2 + 3x - 13$

TRY THESE – Adding and Subtracting Polynomials

1. $(2x - 4y) + (x + 3y)$

2. $(11m^3 + 2m^2 - m) - (-6m^2 + 3m + 4)$

3. $(9a^2 - 3a + 10) + (2a^2 + 5a - 6)$

4. $(2x^2 - 3x + 5) - (4x^2 + 6x - 8)$

5. $(3p^2 - 2p + 1) + (p^2 - 7)$

6. $(7x - 5) - (5x - 3)$

7. $(8b^2 + 5) - (3b^2 + 2b - 9)$

8. $(3d^4 - 2d^3 + 7d) + (9d^4 + 8d^2 - 2d - 12)$

Multiplying Polynomials

When multiplying polynomials, think of it as an extended distributive property. You multiply the term outside the parentheses by all the terms inside the parentheses. If there is more than one term outside the parentheses, then we just have to continue (or extend) the problem. Lastly, simplify by combining any like terms. Sound easy enough? One thing to remember is the properties of exponents. Remember to add the exponents together when you multiply variables with exponents.

Let's try a couple of easy ones.

EXAMPLE 3

Simplify. $(3c)(c+4)$

SOLUTION TO EXAMPLE 3

1.	Distribute the term outside the parentheses to all the terms inside the parentheses.	$3c(c) = 3c^2$ $3c(4) = 12c$
2.	Put it all together.	$3c^2 + 12c$

EXAMPLE 4

Simplify. $5x^3(3x^2 - 2x + 4)$

SOLUTION TO EXAMPLE 4

1.	Distribute the term outside the parentheses to all the terms inside the parentheses.	$5x^3(3x^2) = 15x^5$ $5x^3(-2x) = -10x^4$ $5x^3(4) = 20x^3$
2.	Put it all together.	$15x^5 - 10x^4 + 20x^3$

Let's try a harder problem.

EXAMPLE 5

Simplify. $(3y+2)(2y+4)$

SOLUTION TO EXAMPLE 5

1.	Distribute the term outside the parentheses to all the terms inside the parentheses.	$3y(2y) = 6y^2$ $3y(4) = 12y$ $2(2y) = 4y$ $2(4) = 8$
2.	Combine like terms.	$12y + 4y = 16y$
3.	Put it all together.	$6y^2 + 16y + 8$

<u>**NOTE**</u>: There are two terms in the first parentheses. I had to distribute both term to the terms in the second parentheses. I took the $3y$ and multiply it by the two terms in the second parentheses. Then I took the 2 and multiplied it by the two terms in the second parentheses. Remember to combine any like terms at the end of the problem.

Let's try another one that is a little more difficult.

EXAMPLE 6

Simplify. $(b^2 - 5)(2b^2 - 3b + 6)$

SOLUTION TO EXAMPLE 6

1.	Distribute the term outside the parentheses to all the terms inside the parentheses.	$b^2(2b^2) = 2b^4$ $b^2(-3b) = -3b^3$ $b^2(6) = 6b^2$ $(-5)(2b^2) = -10b^2$ $(-5)(-3b) = 15b$ $(-5)(6) = -30$
2.	Combine like terms.	$6b^2 - 10b^2 = -4b^2$
3.	Put it all together.	$2b^4 - 3b^3 - 4b^2 + 15b - 30$

QUICK NOTE: This problem isn't more difficult. It just has more work to do. Don't associate more work with more difficulty. That adds to math anxiety and we are trying to get rid of that.

And one last one just for fun!

EXAMPLE 7

Simplify. $(4x^2 + 3xy - 2y^2)(2x - 3y)$

SOLUTION TO EXAMPLE 7

1.	Distribute the term outside the parentheses to all the terms inside the parentheses.	$4x^2(2x) = 8x^3$ $4x^2(-3y) = -12x^2y$ $3xy(2x) = 6x^2y$ $3xy(-3y) = -9xy^2$ $(-2y^2)(2x) = -4xy^2$ $(-2y^2)(-3y) = 6y^3$
2.	Combine like terms.	$-12x^2y + 6x^2y = -6x^2y$ $-9xy^2 + (-4xy^2) = -13xy^2$
3.	Put it all together.	$8x^3 - 6x^2y - 13xy^2 + 6y^3$

TRY THESE – Multiplying Polynomials

1. $4x(3x^3 + 2x^2 - 1)$

2. $(2x - 7)(3x - 2)$

3. $(2x^2 - 1)(2x^4 - 3x - 1)$

4. $(3x + 5)(x^4 - 5)$

5. $(7x - 4)(y^2 + 4)$

6. $4xy(2xy^2 + xy - 1)$

7. $(2x^5 + 2x)(5x^4 - 4x + x^8)$

8. $4x(2x^2 + 2x - 4)$

APPENDIX A – MATH STUDY TIPS

We have already discussed some of these tips at the beginning of this study guide. But they are important to know.

GIVE YOURSELF PLENTY OF TIME TO PREPARE FOR THE MATH COMPETENCY EXAM

You really need to allow yourself as much time as possible to effectively prepare for the math competency exam. We recommend at least six weeks. For those who find the material in this guide too easy may not need as much time, but don't kid yourself when it comes to studying the material. If you don't know it, study it.

DON'T BOTHER CRAMMING FOR THE EXAM

Cramming may work for other subject matter, but it doesn't help in math. Your brain needs time to fully grasp the material and cramming overnight does not give it the time. That's why we recommend spending six weeks preparing so each day the brain has a chance to put take the material from its short-term memory and place it in the long-term memory.

PICK A DATE TO TAKE THE MATH COMPETENCY EXAM AND CREATE A SCHEDULE

This may seem obvious, but many students begin studying for the exam and then choose a date when they feel ready. By choosing a date first, it gives you a target and forces you to prepare for it every day. It will also help in creating a study schedule.
Remember to set aside some time every day to go over part of this guide or just the examples. It doesn't need to be hours. Even 20 minutes here and there is better than not doing any math.

MAKE SACRIFICES

This doesn't apply to everyone, but we all have many demands on our everyday life. You might need to DVR your favorite show or cut a few minutes hanging out with friends. Try to understand that this is only temporary. You'll be able to dedicate more time to other things once you take the math competency exam. Decide what you can sacrifice, study hard, pass the exam, and move on to the next stage in your life.

PRACTICE MAKE "MUCH BETTER"

Body builder don't get their muscular bodies by working out once in a while. They are in the gym every day and push themselves even when they are tired. Math is the same way. You need to practice to get better. Get your hands on as many problems as you can and work them out. Work the math department's practice exam several times. Read over this manual several times. The more practice you get, the better you will become and the more confident you will be for the exam.

DON’T USE A CALCULATOR

You won’t be able to use a calculator on the exam. And some of the classes won’t let you use a calculator either. You must learn how to do the problems manually. The problems on the math competency exam were designed so you would not need a calculator. So you need to stop using the calculator as a crutch and prepare for the exam without using it.

A MARATHON, NOT A SPRINT

It will take some time to learn a lot of the material presented to you in this study guide. It is not expected that if you read it once, you’ll be an expert at it right away. Don’t compare yourself with other people. It may seem that other people are picking up on the material really quick while it takes you more time to understand it. Everyone is different and their brains work in different ways. As Aesop said, “Slow and steady wins the race.”

APPENDIX B – EXAM-TAKING STRATEGIES

This section will offer you some basic strategies and tips that are tailored to the math competency exam. The tips in Appendix A are generally applied toward preparation for all mathematics courses.

GET SOME SLEEP THE NIGHT BEFORE THE EXAM

This tip you've probably heard before from another class, professor, friend. Do something relaxing the night before the exam. Watch a movie or your favorite TV show. You can do a few practice problems if you want, but don't make it into a full-blown study session. Cramming the night before isn't going to help! And you probably won't remember any ***new*** concepts you've seen the night before. If you have taken the time to prepare, you wouldn't need to cram. Get enough sleep!

EAT A GOOD BREAKFAST

Even if you normally skip breakfast, you should make time to eat something. Your brain needs energy you get from foods for it to work efficiently. Some good brain-boosting foods include eggs, nuts, yogurt, cottage cheese, fish, blueberries, and sunflower seeds. Some foods to avoid before the exam are chocolates, cookies, cakes and muffins. They tend to send you into a sugar high and end up in a sugar low. And you wouldn't want that to happen in the middle of the exam.

BLOCK OUT OTHERS BEFORE THE EXAM

Whenever you look around at other students just before a test, you see panic. A lot of students are scrambling to cram in those last few concepts. Students get nervous and start discussing the exam with other students. When you start hearing them talk about the test, it is easy for you to get pulled into panic mode also. Now is not the time to be cramming or even panic. Now is the time to relax. Bring an iPod and listen to music. If you have put in the time to study over the past six weeks, there is no need to second-guess yourself.

DON'T SPEND TOO MUCH TIME ON ANY ONE PROBLEM

When you are handed the test, don't always start with Question 1. Skim over the exam, and do the problems you are sure you know what you are doing. You can then go back to the problems you skipped and spend a little more time working them out. I have seen students spend so much time on Question 1, that they end up running out of time and don't finish the exam. If you start to struggle with a problem, move on. But be sure to mark it and return to the problem.

A MARATHON, NOT A SPRINT

The math competency exam is a timed test. As other students are standing up and turning in their exams, don't worry about it. It is not a race to see who turns in their exam first. It is about how well you do on the problems. Take your time. Be confident with your answers. As Aesop said, "Slow and steady wins the race."

APPENDIX C – ANSWERS TO "TRY THESE" PROBLEMS

TRY THESE – Estimating (pg. 11)

1. 700
2. 8,000
3. 1,540
4. 7,900
5. C
6. B
7. A
8. a) about 500 people

   b) No. There are only about 500 seats. So 350 students won't have a seat.

TRY THESE – Order of Operations (pg. 14)

1. 54
2. $45\frac{1}{2}$
3. -343
4. -190
5. $3\frac{1}{9}$
6. $1\frac{69}{100}$

TRY THESE – Multiplying and Dividing of Fractions (pg. 20)

1. $\frac{2}{3}$
2. $\frac{12}{25}$
3. $26\frac{2}{3}$
4. $1\frac{46}{125}$
5. 20 sashes

TRY THESE – Adding and Subtracting Fractions (pg. 26)

1. $1\frac{23}{36}$
2. $19\frac{37}{40}$
3. $\frac{1}{21}$
4. $6\frac{7}{20}$
5. $\frac{13}{15}$
6. $8\frac{28}{45}$
7. $3\frac{11}{16}$ ft. left over from the board

TRY THESE – Properties of Exponents (pg. 28)

1. $6xy^2z^3$
2. $\frac{5a}{4}$
3. $\frac{16x^{10}}{y^4}$
4. $8x^9y^{15}z^3$
5. $\frac{2b}{3a}$
6. $-4qr^6s^{10}$
7. $18x^5$
8. $\frac{9x^{22}y^8}{25z^4}$
9. $\frac{1z^9}{3y}$

TRY THESE – Solving Equations (pg. 34)

1. $x = -11$
2. $x = 0$
3. $x = -4$
4. $x = -\frac{2}{7}$
5. $x = 1\frac{7}{12}$
6. $R = -L + 1$
7. $Q = \frac{16R-4}{7}$
8. $x = 1$
9. $V = R - 3$
10. $x = 18$

TRY THESE – Solving Inequalities (pg. 36)

1. $x \leq -48$
2. $x > -10$
3. $x \leq 2$
4. $x > -\frac{3}{10}$
5. $x < -\frac{5}{2}$
6. $x \geq -10$

TRY THESE – Solving Equations (pg. 34)

1. $h = 4$
2. $x = 40$
3. $m = 52$
4. $g = 13$
5. $y = 1.2$
6. $a = 15$
7. $t = 198.072$ minutes (about 3 hours 18 minutes)

TRY THESE – Percentages (pg. 44)

1. 16
2. 20
3. 75%
4. $16\frac{2}{3}$
5. 20%
6. Adams Field has a capacity of 6,000 of seats.

TRY THESE – Solving Equations (pg. 34)

1. $x = -11$
2. $x = 0$
3. $x = -4$
4. $x = -\frac{2}{7}$
5. $x = 1\frac{7}{12}$
6. $R = -L + 1$
7. $Q = \frac{16R-4}{7}$
8. $x = 1$
9. $V = R - 3$
10. $x = 18$

TRY THESE – Finding Slope and y-Intercept (pg. 49)

1. $-\frac{9}{1}$
2. Undefined
3. 0
4. $\frac{1}{10}$
5. Slope is 1; y-intercept is the point (0, -3)
6. Slope is $-\frac{4}{3}$; y-intercept is the point (0, -4)
7. Slope is undefined; there is no y-intercept because it is a vertical line.
8. Slope is 3; y-intercept is the point (0, 4)

TRY THESE – Graphing Linear Equations (pg. 51)

1. Slope is -3; y-intercept is the point $(0, 1)$
2. Slope is $undefined$; No y-intercept because it is a vertical line.
3. Slope is $\frac{2}{5}$; y-intercept is the point $(0, -2)$
4. Slope is 0; y-intercept is the point $(0, -2)$
5. Slope is $\frac{1}{2}$; y-intercept is the point $(0, -3)$
6. Slope is -1; y-intercept is the point $(0, 2)$
7. Slope is $undefined$; No y-intercept because it is a vertical line.
8. Slope is $\frac{3}{7}$; y-intercept is the point $(0, -1)$

TRY THESE – Graphing Linear Equations (pg. 53)

1. A little more than 7
2. 11
3. 7
4. A little more than 2
5. A little under 10
6. A little under 6
7. A little more than 12
8. $\frac{4}{5}$

TRY THESE – Adding and Subtracting Polynomials (pg. 55)

1. $3x - 1y$
2. $11m^3 + 8m^2 - 4m - 4$
3. $11a^2 + 2a + 4$
4. $-2x^2 - 9x + 13$
5. $4p^2 - 2p - 6$
6. $2x - 2$
7. $5b^2 - 2b + 14$
8. $12d^4 - 2d^3 + 8d^2 + 5d - 12$

TRY THESE – Multiplying Polynomials (pg. 58)

1. $12x^4 + 8x^3 - 4x$
2. $6x^2 - 25x + 14$
3. $4x^6 - 2x^4 - 6x^3 - 2x^2 + 3x + 1$
4. $3x^5 + 5x^4 - 15x - 25$
5. $7xy^2 + 28x - 4y^2 - 16$
6. $8x^2y^3 + 4x^2y^2 - 4xy$
7. $2x^{13} + 12x^9 - 8x^6 + 10x^5 - 8x^2$
8. $8x^3 + 8x^2 - 16x$