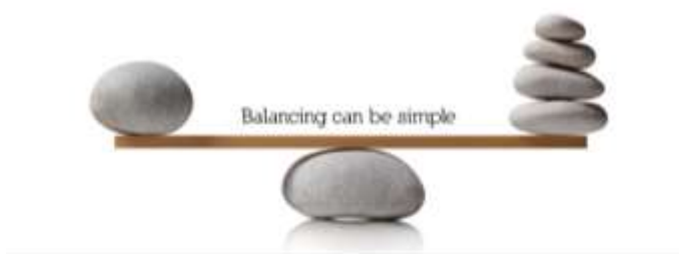
Chapter 6: Solving Equations

Solving equations is a skill that is tested many times on WSU's Math Competency Exam. Additionally, many of the problems that ask you to solve an equation will invariably involve many of the other skills discussed in the other chapters in this study guide. You will also find the solving equations will be found in all math courses if it is MAT 0993 or MAT 7000. Knowing how to break down an equation and solve for a variable is inevitable to succeed.

One important idea to remember about equations is that two expressions are equal. I always imagine a seesaw (or some people call it a teeter-totter). If the two people on either side are the same weight, the seesaw is balanced (or equal).

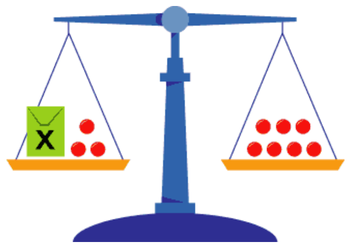


But what happens when one side is heavier than the other? You guessed it, one side goes up and the other side goes down. The seesaw is now out of balance and no longer equal. To get it back to being equal, we need to either remove something from the heavier side or add more to the lighter side.

That is what an equation is all about…keeping both sides equal. So how do we do this in mathematics? Good question. Let's see.

When solving an equation we have one ultimate goal in mind. That goal is to "Isolate the variable". The variable is the "unknown" and could be represented by a number of different letters like x, y, z or a to name a few. Let us look at a rather simple example.

EXAMPLE 1
Solve: $x + 3 = 7$



I have included a picture to visualize the equation. Notice that the balance is equal. What would happen if I took one of the red marbles from the left side? The right side would be heavier and drop down. To balance it out, I need to take one red marble from the right. This brings us to a couple of rules of equations.

RULE 1: You may add (or subtract) the same number from both sides of the equal sign.

RULE 2: You may multiply (or divide) the same number from both sides of the equal sign.

The overall rule I like to remember is "***What you do to one side of the equal sign, you must do the same thing to the other side***".

SOLUTION TO EXAMPLE 1
Remember our goal is to get the variable, x, by alone.

1.	$x + 3 = 7$	Start with our equation. The variable is x. To get x by itself, we have to get rid of the "+ 3".
2.	$x + 3 - 3 = 7 - 3$	To get rid of the "+ 3" we subtract 3 from both sides.
3.	$x = 4$	Simplify.

When we say "get rid of" something we don't mean simply delete it from the problem. We must do the opposite operation to whatever we are trying to "get rid of", but we do that operation to BOTH sides of the equal sign to ensure things stay balanced. Combining numbers is not the same as "getting rid of" them. In the example above we combine "7 – 3" to equal "4". We combined the two numbers together.

Let's take a look at another example.

EXAMPLE 2

Solve: $2x + 4 = 10$

SOLUTION TO EXAMPLE 2

Remember our goal is to get the variable, x, by alone.

1.	$2x + 4 = 10$	
2.	$2x + 4 - 4 = 10 - 4$ $2x = 6$	To get rid of the "+ 4" we subtract 4 from both sides.
3.	$\frac{2x}{2} = \frac{6}{2}$	To get rid of the "2 times", we need to divide both sides by 2.
4.	$x = 3$	Simplify.

The beautiful thing about solving equations is that you can always check to see if you came up with the correct answer. To check if "3" is the correct answer, simply "plug" 3 back into the original equation for x and see if it makes sense.

5.	$2(3) + 4 \stackrel{?}{=} 10$	
6.	$6 + 4 \stackrel{?}{=} 10$	
7.	$10 = 10$	Since both sides are equal, $x = 3$ is the correct answer.

As you may think, equations they'll ask you to solve on the math competency exam aren't quite as simple, but the general method we used above works on virtually all of the problems you'll encounter. They will likely be a little more complex.

Let's look at a problem taken directly from the practice exam.

EXAMPLE 3
Solve: $3(2-y)-4=4-5(y+1)$

SOLUTION TO EXAMPLE 3
Even though it looks a little more involved than the other examples we've seen, it isn't anything we cannot handle. But where do we begin?

1.		$3(2-y)-4=4-5(y+1)$
2.	Distribute the numbers in front of the parentheses.	$3(2)-3(y)-4=4-5(y)+(-5)(1)$ $6-3y-4=4-5y-5$
3.	Combine like terms on each side of the equal sign.	$6-4-3y=4-5-5y$ $2-3y=-1-5y$
4.	Notice that there are variables on both sides of the equal sign. We need to get them both on the same side. It does not matter which side. In this case, I will get rid of the "$-5y$" from both sides.	$2-3y+5y=-1-5y+5y$ $2+2y=-1$
5.	Now we still have to get the "y" variable by itself. Let's get rid of the 2	$2-2+2y=-1-2$ $2y=-3$
6.	Lastly, we need to divide both sides by 2.	$\frac{2y}{2}=\frac{-3}{2}$ $y=\frac{-3}{2}$
7.	Remember to plug the number back into the original equation to check if it is reasonable.	$3\left(2-\frac{-3}{2}\right)-4=4-5\left(\frac{-3}{2}+1\right)$

In high school, most equations that you solved might have a nice whole number as an answer. Most students expect that when solving equations. Notice that the answer for EXAMPLE 3 was a fraction. Just because you get a fraction does not mean you got an incorrect answer. Be confident with your steps and you'll do fine.

Let's take a look at one last type of equation you might be asked to solve on the math competency exam. In EXAMPLE 4, you will be asked to solve for a variable. Don't be afraid of this type of equation. You follow the same procedures we have worked with in other examples. The only difference is we are using mainly variables instead of actual numbers.

EXAMPLE 4

Solve for T: $V = \frac{2P+T}{T}$

SOLUTION TO EXAMPLE 4

Even though it looks a little more involved than the other examples we've seen, it isn't anything we cannot handle. But where do we begin?

1.		$V = \frac{2P+T}{T}$
2.	Remember a fraction is another way to write a division problem. So I am going to switch this to a division before moving on.	$V = (2P+T) \div T$
3.	Now we can get rid of the "$\div T$" by multiplying both sides by T.	$V \cdot T = (2P+T) \div T \cdot T$ $V \cdot T = 2P + T$
4.	The next step would be to get the T's on one side of the equal sign. So I am going to subtract the T from both sides. We cannot combine the "$V \cdot T - T$" since there is a multiplication and an addition sign. So be careful here!	$V \cdot T - T = 2P + T - T$ $V \cdot T - T = 2P$
5.	What we can do though is to use the distributive property. This allows us to "divide" or "factor out" the T from the expression.	$T(V-1) = 2P$
6.	We have one last operation to do. To get the T by itself, we need to divide the parentheses from both sides.	$\frac{T(V-1)}{(V-1)} = \frac{2P}{(V-1)}$
7.	So our final answer will be this equation.	$T = \frac{2P}{(V-1)}$

In the other examples, we told you to plug the answer back into the original equation to see if it makes sense. But in this case, we don't just have one numeric value we can plug into the problem. We have this expression. If you are ambitious, you can plug it in and see if it works out. But I would suggest going over step-by-step to make sure your steps are algebraically sound.

TRY-THESE – Solving Equations

1. $4(2x+3)=7x+1$

2. $3x-12=-12+6x$

3. $\frac{1}{3}x-\frac{1}{4}=\frac{5}{12}+\frac{1}{2}x$

4. $3(6x-2)+2=2(2x-4)$

5. $\frac{2}{3}(6x-2)=5$

6. Solve for R: $2L-4R+2=4L-2R$

7. Solve for Q: $\frac{Q-4}{4}=2Q-4R$

8. $3x-(4x-3)+2=2(4x-2)$

9. $Solve\ for\ V$: $-2(V-3)+4V=2R$

10. $2x-3+\frac{1}{2}x=2x+6$