# Chapter 4: Fractions

There are several operations you are expected to understand and perform on the competency exam. You will probably not see a problem that specifically asks you to add, subtract, multiply or divide two fractions. Instead you might encounter more difficult problems that include working with fractions (as in Problems 5 and 6 from Chapter 3: Try-These).

Let's think about some observations about fractions.

1. Fractions are basically the same as decimals.
2. Fractions are division problems in disguise.
3. Dividing and multiplying fractions are slight easier than adding and subtracting fractions.
4. When multiplying or dividing fractions, you do not need a common denominator.
5. **When adding or subtracting fractions, a common denominator is needed**.

Fractions represent equal parts of a whole. When you see a fraction like $\frac{2}{3}$, the number on top is called the **numerator**. It represents the number of parts being described. The number on the bottom is called the **denominator**. The denominator represents the total number of parts that make up one whole.

So $\frac{2}{3}$ means that there are 2 equal parts out of 3 parts. Visually, $\frac{2}{3}$ would look like

## Equivalent Fractions

Any fractions that represent the same amount are called **equivalent fractions**. An equivalent fraction can be found by multiplying or dividing both the numerator and denominator by the same non-zero number.

**EXAMPLE 1**

Find four equivalent fractions of $\frac{2}{5}$.

**SOLUTION TO EXAMPLE 1**

| | | | |
|---|---|---|---|
| $\frac{2\times 2}{5\times 2}=\frac{4}{10}$ | $\frac{2\times 3}{5\times 3}=\frac{6}{15}$ | $\frac{2\times 4}{5\times 4}=\frac{8}{20}$ | $\frac{2\times 25}{5\times 25}=\frac{50}{125}$ |

**NOTE**: In the last solution, I used 25. I just wanted to show that we can multiply by any number we want to get an equivalent fraction.

The term "simplify" refers to reducing a fraction down to the lowest terms. When a fraction is simplified or reduced to lowest terms, there is no one number that can be evenly divided into *both* the numerator and denominator (other than 1).

**EXAMPLE 2**

Simplify $\frac{24}{42}$ to its lowest term.

**SOLUTION TO EXAMPLE 2**

Some of you might remember that you have to find the Greatest Common Factor (GCF) to divide the numerator and denominator. The GCF is great, but it not always the quickest to find the GCF. Just keep dividing out factors that you can divide into the numerator and denominator. It might take you a couple of extra steps, but you are still finding the answer.

| | |
|---|---|
| $\frac{24 \div 2}{42 \div 2} = \frac{12}{21}$ | $\frac{12 \div 3}{21 \div 3} = \frac{4}{7}$ |

Since there are no values that divide into 4 and into 7 evenly, we found our fraction in lowest terms.

The GCF of 24 and 42 is 6. So if we divide 6 from the numerator and denominator, we get the same answer.

$$\frac{24 \div 6}{42 \div 6} = \frac{4}{7}$$

**NOTE**: To save yourself some time when simplifying fractions, divide out larger numbers from the numerator and denominator.

**No matter where you start, just remember that the exam's final answer will always be the most simplified.**

## Mixed Numbers and Improper Fractions

There are three types of fractions that can be written. The first is the **proper fraction**. These are the most common fractions that we see. A proper fraction is where the numerator is smaller than the denominator. Some examples of proper fractions are $\frac{1}{2}$, $\frac{3}{8}$, $\frac{10}{27}$, and $\frac{1000}{2589}$.

Another type of fraction is the **improper fraction**. An improper fraction has the numerator the same value or greater than the denominator. So a couple of examples of improper fractions are $\frac{3}{2}$, $\frac{12}{5}$, $\frac{15}{15}$, and $\frac{104}{26}$. Notice that even if the numerator and denominator are the same, it is still an improper fraction.

The last type of fraction is called the **mixed fraction** (or **mixed number**). A mixed fraction is a mixture of a whole number value and a fraction. A few examples of mixed numbers are $8\frac{1}{2}$, $10\frac{2}{7}$, and $93\frac{3}{4}$.

## Converting Mixed Fractions into Improper Fractions

- Multiply the whole number part by the fraction's denominator.
- Add it to the numerator.
- Write that result on top of the original denominator

**EXAMPLE 3**

Write $2\frac{3}{10}$ as an improper fraction.

**SOLUTION TO EXAMPLE 3**

| | | |
|---|---|---|
| 1. | Multiply the whole number (2) by the fraction's denominator (10). | $2 \times 10 = 20$ |
| 2. | Add it (20) to the numerator (3) | $20 + 3 = 23$ |
| 3. | Place over the original denominator (10). | $\frac{23}{10}$ |

## Converting Improper Fractions to Mixed Numbers

- Divide the numerator by the denominator.
- Write down the whole number answer.
- Place the remainder over the original denominator.

**EXAMPLE 4**

Write $\frac{11}{4}$ as a mixed number.

**SOLUTION TO EXAMPLE 4**

| | | |
|---|---|---|
| 1. | Divide numerator (11) by denominator (4). | $4\overline{)\,11}$ (2), $-8$, $3$ |
| 2. | Write down the whole number. | 2 |
| 3. | Place the remainder (3) over the original denominator (4). | $2\frac{3}{4}$ |

## Multiplying Fractions

Most people think we should always do adding or subtracting before multiplication or division. In the case of fractions, multiplying and dividing of fractions are slightly easier than adding and subtracting.

Before we get into how to multiply fractions, first we have to make sure all the numbers are written as fractions (either proper or improper fractions). We cannot have any mixed numbers. If you do not remember how to convert mixed numbers to improper fractions, please review the previous section of this chapter.

Now that we have all fractions, let's multiply our fractions. Multiply the numerators together. Then multiply the denominators together. And simplify the result if you can. That's it!

**EXAMPLE 5**

Multiply $\frac{3}{4} \times \frac{2}{6}$.

**SOLUTION TO EXAMPLE 5**

| | | |
|---|---|---|
| 1. | Multiply the numerators. | $3 \times 2 = 6$ |
| 2. | Multiply the denominators. | $4 \times 6 = 24$ |
| 3. | Simplify the fraction if you can | $\frac{6 \div 6}{24 \div 6} = \frac{1}{4}$ |

**EXAMPLE 6**

Multiply $2\frac{1}{3} \times 4$.

**SOLUTION TO EXAMPLE 6**

| | | |
|---|---|---|
| 1. | Make sure numbers are written as proper or improper fractions. You can change any whole number into an improper fraction by placing a 1 in the denominator. | $\frac{7}{3} \times \frac{4}{1}$ |
| 2. | Multiply numerators together. | $7 \times 4 = 28$ |
| 3. | Multiply denominators together. | $3 \times 1 = 3$ |
| 4. | Simplify the fraction if you can. Notice that the fraction is an improper fraction. Convert it to a mixed number. | $\frac{28}{3} = 9\frac{1}{3}$ |

## Dividing Fractions

Dividing fractions is just as easy as multiplying fractions. In fact, almost all the steps are the same. But there is one extra step that we need to discuss before we can divide. That is the "**reciprocal**".

The reciprocal switches the values of the numerator and denominator. Many people think of it as "flipping" the fraction upside-down. But remember it has to be a fraction (proper or improper) and not a mixed number. Let's see how that works.

**EXAMPLE 7**

Find the reciprocal of $\frac{5}{9}$.

**SOLUTION TO EXAMPLE 7**

| | | |
|---|---|---|
| 1. | Switch the numbers in the numerator and denominator. | $\frac{9}{5}$ |

Don't worry about simplifying right now. We will eventually have to, but not until the end of the entire problem.

Why did we have to know about the reciprocal? Because dividing fractions is the same as "multiplying the first fraction by its reciprocal". Now that we make it a multiplication problem, we use the same step we used for Examples 5 and 6. So let's see a few problems.

**EXAMPLE 8**

Divide $\frac{3}{4} \div \frac{2}{6}$.

**SOLUTION TO EXAMPLE 8**

| | | |
|---|---|---|
| 1. | Multiply by the reciprocal.<br><br>**NOTE**: We do not take the reciprocal of the first fraction. The first fraction stays the same. When dividing, we take the reciprocal of the second fraction only! | $\frac{3}{4} \times \frac{6}{2}$ |
| 2. | Multiply the numerators | $3 \times 6 = 18$ |
| 3. | Multiply the denominators. | $4 \times 2 = 8$ |
| 4. | Simplify if you can. | $\frac{18}{8} = \frac{9}{4} = 2\frac{1}{4}$ |

**EXAMPLE 9**

Divide $2\frac{1}{3} \div 4$.

**SOLUTION TO EXAMPLE 9**

| | | |
|---|---|---|
| 1. | Make sure numbers are written as proper or improper fractions. You can change any whole number into an improper fraction by placing a 1 in the denominator. | $\frac{7}{3} \div \frac{4}{1}$ |
| 2. | Multiply by the reciprocal. | $\frac{7}{3} \times \frac{1}{4}$ |
| 3. | Multiply numerators together. | $7 \times 1 = 7$ |
| 4. | Multiply denominators together. | $3 \times 4 = 12$ |
| 5. | Simplify the fraction if you can. In this case, we can't so we don't do anything. | $\frac{7}{12}$ |

## TRY THESE – Multiplying and Dividing of Fractions

Perform the given operation. Be sure your answer is in lowest terms.

1. $\frac{3}{4} \times \frac{8}{9}$

2. $\frac{3}{10} \div \frac{5}{8}$

3. $5\frac{5}{7} \times 4\frac{2}{3}$

4. $3\frac{4}{5} \div 2\frac{7}{9}$

5. Chris bought 15 yds of fabric from which to cut sashes for the dance team. If each sash is $\frac{3}{4}$ yds longs, how many sashes can she cut?

## Adding Fractions

If you recall from your math classes from way back when, you'll remember that you can only add (or subtract) two fractions if they have the same denominator. Most likely, the problems you'll encounter on the competency exam will require you to find a common denominator so you can perform the operation.

There are many ways to find a common denominator. I am going to show you the easiest one to learn, but this method may also require a little more work at the end in terms of simplifying your final answer.

The easiest way to find the common denominator is to simply multiply both denominators together. But remember if you multiply the denominator by a number, you have to multiply the numerator by the same number to keep it as an equivalent fraction. (See the previous section regarding equivalent fractions on page 13 if you forgot how to do this already.) Let's see a couple of examples finding common denominators.

**EXAMPLE 10**

Re-write the problem $\frac{2}{3}+\frac{1}{5}$ using common denominators. We do not need to perform the operation at this time.

**SOLUTION TO EXAMPLE 10**

| | | |
|---|---|---|
| 1. | Multiply the two denominators together. This will be my common denominator. | $3\times 5=15$ |
| 2. | Now we have to find the equivalent fraction for $\frac{2}{3}$ but with 15 as the denominator. In this case multiply the numerator and denominator by 5. | $\frac{2\times 5}{3\times 5}=\frac{10}{15}$ |
| 3. | Now we have to find the equivalent fraction for $\frac{1}{5}$ but with 15 as the denominator. In this case multiply the numerator and denominator by 3. | $\frac{1\times 3}{5\times 3}=\frac{3}{15}$ |
| 4. | When we re-write the problem, we get | $\frac{10}{15}+\frac{3}{15}$ |

At this point we are done. But we will talk about how to add fractions after we get those common denominators! So just hang on for a minute.

**EXAMPLE 11**

Re-write the problem $\frac{3}{4}-\frac{2}{7}$ using common denominators. We do not need to perform the operation at this time. (I know this section is on adding fractions, but I thought I would put in a subtraction problem here since finding common denominators for subtraction is the same concept.)

**SOLUTION TO EXAMPLE 11**

| | | |
|---|---|---|
| 1. | Multiply the two denominators together. This will be my common denominator. | $4 \times 7 = 28$ |
| 2. | Now we have to find the equivalent fraction for $\frac{3}{4}$ but with 28 as the denominator. In this case multiply the numerator and denominator by 7. | $\frac{3 \times 7}{4 \times 7} = \frac{21}{28}$ |
| 3. | Now we have to find the equivalent fraction for $\frac{2}{7}$ but with 28 as the denominator. In this case multiply the numerator and denominator by 4. | $\frac{2 \times 4}{7 \times 4} = \frac{8}{28}$ |
| 4. | When we re-write the problem, we get | $\frac{21}{28}+\frac{8}{28}$ |

Okay. Let's move on to adding fractions with common denominators. This is the easy part. We've already done the hard work. To add fractions with common denominators, add the two numerators together. Then place it over the common denominator. Simplify your fraction if necessary, especially if you need to convert it to a mixed number!

Now let's finish the problems we started.

**EXAMPLE 10 (revisited)**

Add $\frac{2}{3}+\frac{1}{5}$.

**SOLUTION TO EXAMPLE 10 (revisited)**

We don't need to show all the work of finding the common denominator since we've already done it on the previous page.

| | | |
|---|---|---|
| 1. | When we re-write the problem, we get | $\frac{10}{15}+\frac{3}{15}$ |
| 2. | Add the numerators. | $10 + 3 = 13$ |
| 3. | Place over the common denominator. | $\frac{13}{15}$ |
| 4. | Simplify. In this case, it is in lowest terms already. | |

**EXAMPLE 12**

Add $2\frac{3}{5}+4\frac{2}{9}$.

**SOLUTION TO EXAMPLE 12**

| | | |
|---|---|---|
| 1. | Multiply the two denominators together.<br>This will be my common denominator. | $5\times 9=45$ |
| 2. | Now we have to find the equivalent fraction for $\frac{3}{5}$ but with 45 as the denominator. In this case multiply the numerator and denominator by 9. | $\frac{3\times 9}{5\times 9}=\frac{27}{45}$ |
| 3. | Now we have to find the equivalent fraction for $\frac{2}{9}$ but with 45 as the denominator. In this case multiply the numerator and denominator by 5. | $\frac{2\times 5}{9\times 5}=\frac{10}{45}$ |
| 4. | When we re-write the problem, we get | $2\frac{27}{45}+4\frac{10}{45}$ |
| 5. | Add numerators. | $27+10=37$ |
| 6. | Place over the common denominator and simplify. | $\frac{37}{45}$ |
| 7. | Add the whole numbers. | $2+4=6$ |
| 8. | Put it all together. | $6\frac{37}{45}$ |

**EXAMPLE 13**

Add $5\frac{1}{4}+19\frac{11}{12}$.

**SOLUTION TO EXAMPLE 13**

| | | |
|---|---|---|
| 1. | Multiply the two denominators together.<br>This will be my common denominator. | $4\times 12=48$ |
| 2. | Now we have to find the equivalent fraction for $\frac{1}{4}$ but with 48 as the denominator. In this case multiply the numerator and denominator by 12. | $\frac{1\times 12}{4\times 12}=\frac{12}{48}$ |
| 3. | Now we have to find the equivalent fraction for $\frac{11}{12}$ but with 48 as the denominator. In this case multiply the numerator and denominator by 4. | $\frac{11\times 4}{12\times 4}=\frac{44}{48}$ |
| 4. | When we re-write the problem, we get | $5\frac{12}{48}+19\frac{44}{48}$ |
| 5. | Add numerators. | $12+44=56$ |
| 6. | Place over the common denominator and simplify. | $\frac{56}{48}=\frac{7}{6}=1\frac{1}{6}$ |
| 7. | Add the whole numbers.<br>Notice we added the 1 whole number from Step #6 above since it was a mixed number. | $5+19+1=25$ |
| 8. | Put it all together. | $25\frac{1}{6}$ |

## Subtracting Fractions

Many of the steps for subtracting fractions are similar to those on adding fractions.
In general, the steps for subtracting fractions would be:

1. Find common denominators.
2. Re-write each fraction using the common denominators.
3. Subtract the numerators and place of the common denominator.
4. And simplify if we can.

So let's take a look at a problem and review our step, but this time we are going to subtract fractions.

**EXAMPLE 14**

Subtract $5\frac{3}{4} - 1\frac{5}{12}$.

**SOLUTION TO EXAMPLE 14**

| | | |
|---|---|---|
| 1. | Multiply the two denominators together.<br>This will be my common denominator. | $4 \times 12 = 48$ |
| 2. | Now we have to find the equivalent fraction for $\frac{3}{4}$ but with 48 as the denominator. In this case multiply the numerator and denominator by 12. | $\frac{3 \times 12}{4 \times 12} = \frac{36}{48}$ |
| 3. | Now we have to find the equivalent fraction for $\frac{5}{12}$ but with 48 as the denominator. In this case multiply the numerator and denominator by 4. | $\frac{5 \times 4}{12 \times 4} = \frac{20}{48}$ |
| 4. | When we re-write the problem, we get | $5\frac{36}{48} - 1\frac{20}{48}$ |
| 5. | Subtract numerators. | $36 - 20 = 16$ |
| 6. | Place over the common denominator and simplify. | $\frac{16}{48} = \frac{1}{3}$ |
| 7. | Subtract the whole numbers. | $5 - 1 = 4$ |
| 8. | Put it all together. | $4\frac{1}{3}$ |

Great! Now for the catch! I know. There's always a catch. What happens when the first fraction is smaller than the second fraction? What do we do? Panic? Of course not. Let's take a look at a problem and work out the steps.

**EXAMPLE 15**

Subtract $8\frac{1}{3} - 6\frac{4}{5}$.

**SOLUTION TO EXAMPLE 13**

| | | |
|---|---|---|
| 1. | Multiply the two denominators together. This will be my common denominator. | $3 \times 5 = 15$ |
| 2. | Now we have to find the equivalent fraction for $\frac{1}{3}$ but with 15 as the denominator. In this case multiply the numerator and denominator by 5. | $\frac{1 \times 5}{3 \times 5} = \frac{5}{15}$ |
| 3. | Now we have to find the equivalent fraction for $\frac{4}{5}$ but with 15 as the denominator. In this case multiply the numerator and denominator by 3. | $\frac{4 \times 3}{5 \times 3} = \frac{12}{15}$ |
| 4. | When we re-write the problem, we get | $8\frac{5}{15} - 6\frac{12}{15}$ |
| 5. | Subtract numerators. | $5 - 12 = -7$ |
| 6. | Place over the common denominator and simplify. | $\frac{-7}{15}$ |
| 7. | Subtract the whole numbers. | $8 - 6 = 2$ |
| 8. | Put it all together. | $2 - \frac{7}{15}$ |

At this point, I am going to stop the problem and talk about "regrouping". Regrouping is a method of changing a mixed number (or whole number) into a fraction with a whole number PLUS an improper fraction. We only use this method to simplify subtraction problems.
In order to regroup a fraction, we also start by saying it is "one less than our number" + 1. What? Don't worry it sounds worse than it actually is.

| | | |
|---|---|---|
| 9. | Regroup the whole number. The first number is going to be our whole number. The "+ 1" we are going to re-write it as "+ fraction with the common denominator". Recall that 1 is just the number over itself. In our case, the number is 15. | $2 = 1 + 1$<br>$2 = 1 + \frac{15}{15}$ |
| 10. | Re-write our problem with the regrouping would look like this. | $1 + \frac{15}{15} - \frac{7}{15}$ |
| 11. | Subtract the fractions and simplify again if necessary. | $1 + \left(\frac{15}{15} - \frac{7}{15}\right)$<br>$1 + \left(\frac{8}{15}\right)$ |
| 12. | Put it all together. | $1\frac{8}{15}$ |

## TRY THESE – Adding and Subtracting Fractions

Perform the given operation. Be sure your answer is in lowest terms.

1. $\frac{3}{4}+\frac{8}{9}$

2. $7\frac{3}{10}+12\frac{5}{8}$

3. $\frac{5}{7}-\frac{2}{3}$

4. $8\frac{3}{5}-2\frac{1}{4}$

5. $5\frac{7}{10}-4\frac{5}{6}$

6. $13\frac{2}{5}-4\frac{7}{9}$

7. Joe has a piece of board that is 8 ft. long. He cuts off a piece of the board that is $4\frac{5}{16}$ ft. in length. How much of the board does Joe have left?