

CHAPTER 13: POLYNOMIALS

We will not cover all there is to know about polynomials for the math competency exam. We will go over the addition, subtraction, and multiplication of polynomials. We will not go over factoring in this study guide. We will go over it in the study guide for PART B.

So let's begin with a few definitions that will help us understand polynomials.

A **term** (or **monomial**) is an expression that is either a single number, a variable, or the product of a number and one or more variables. So examples of terms are 4, x , $3y$, $2y^2$, $5xy$, $6x^3y^2$.

A **polynomial** is the sum of two or more monomials. Each monomial is called a **term** of the polynomial and are separated by either a "+" or "-" sign.

A polynomial is written in **standard form** when its terms are arranged in order from the greatest or least powers of one of the variables.

Like terms are terms that have exactly the same variable combination (exponents and all). Some examples of like terms are x^2y , $5x^2y$, and $-3x^2y$. Notice that each of the terms have an "x" variable with a power of 2 and a "y" variable.

These are NOT like terms: $xy, x^2y, 2xy^2$

Even though they all have an "x" variable and a "y" variable, their exponents aren't always the same.

We really need to pay attention to which terms are like terms when we add or subtract polynomials.

Adding and Subtracting Polynomials

To add polynomials, we combine the like terms together. That's why it's important to know about the like terms.

EXAMPLE 1

Simplify. $(4x^2 + 3x - 2) + (3x^2 + 7)$

SOLUTION TO EXAMPLE 1

1.	Add the like terms together.	$(4x^2 + 3x - 2) + (3x^2 + 7)$ $(4x^2 + 3x^2) + 3x + (-2 + 7)$
2.	Simplify.	$(4x^2 + 3x^2) = 7x^2$ $(-2 + 7) = 5$
3.	Put it all together.	$7x^2 + 3x + 5$

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NOTE: When I added the like terms, I only added the coefficients (the numbers in front of the variable). The variable and exponent stayed the same. Remember the property of exponents. You only add the exponents together when you multiply.

Subtraction of polynomials is very similar to adding of real numbers. To subtract one polynomial from another, add its opposite and simplify.

EXAMPLE 2

Simplify. $(5x^2 - 11) - (3x^2 - 3x + 2)$

SOLUTION TO EXAMPLE 2

1.	Subtract the like terms together.	$(5x^2 - 11) - (3x^2 - 3x + 2)$ $(5x^2 - 3x^2) + (0x - (-3x)) + ((-11) - 2)$
2.	Simplify.	$(5x^2 - 3x^2) = 2x^2$ $(0x - (-3x)) = 0x + 3x = 3x$ $((-11) - 2) = -13$
3.	Put it all together.	$2x^2 + 3x - 13$

TRY THESE – Adding and Subtracting Polynomials

1. $(2x - 4y) + (x + 3y)$

2. $(11m^3 + 2m^2 - m) - (-6m^2 + 3m + 4)$

3. $(9a^2 - 3a + 10) + (2a^2 + 5a - 6)$

4. $(2x^2 - 3x + 5) - (4x^2 + 6x - 8)$

5. $(3p^2 - 2p + 1) + (p^2 - 7)$

6. $(7x - 5) - (5x - 3)$

7. $(8b^2 + 5) - (3b^2 + 2b - 9)$

8. $(3d^4 - 2d^3 + 7d) + (9d^4 + 8d^2 - 2d - 12)$

Multiplying Polynomials

When multiplying polynomials, think of it as an extended distributive property. You multiply the term outside the parentheses by all the terms inside the parentheses. If there is more than one term outside the parentheses, then we just have to continue (or extend) the problem. Lastly, simplify by combining any like terms. Sound easy enough? One thing to remember is the properties of exponents. Remember to add the exponents together when you multiply variables with exponents.

Let's try a couple of easy ones.

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EXAMPLE 3

Simplify. $(3c)(c + 4)$

SOLUTION TO EXAMPLE 3

1.	Distribute the term outside the parentheses to all the terms inside the parentheses.	$3c(c) = 3c^2$ $3c(4) = 12c$
2.	Put it all together.	$3c^2 + 12c$

EXAMPLE 4

Simplify. $5x^3(3x^2 - 2x + 4)$

SOLUTION TO EXAMPLE 4

1.	Distribute the term outside the parentheses to all the terms inside the parentheses.	$5x^3(3x^2) = 15x^5$ $5x^3(-2x) = -10x^4$ $5x^3(4) = 20x^3$
2.	Put it all together.	$15x^5 - 10x^4 + 20x^3$

Let's try a harder problem.

EXAMPLE 5

Simplify. $(3y + 2)(2y + 4)$

SOLUTION TO EXAMPLE 5

1.	Distribute the term outside the parentheses to all the terms inside the parentheses.	$3y(2y) = 6y^2$ $3y(4) = 12y$ $2(2y) = 4y$ $2(4) = 8$
2.	Combine like terms.	$12y + 4y = 16y$
3.	Put it all together.	$6y^2 + 16y + 8$

NOTE: There are two terms in the first parentheses. I had to distribute both term to the terms in the second parentheses. I took the $3y$ and multiply it by the two terms in the second parentheses. Then I took the 2 and multiplied it by the two terms in the second parentheses. Remember to combine any like terms at the end of the problem.

Let's try another one that is a little more difficult.

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EXAMPLE 6

Simplify. $(b^2 - 5)(2b^2 - 3b + 6)$

SOLUTION TO EXAMPLE 6

1.	Distribute the term outside the parentheses to all the terms inside the parentheses.	$b^2(2b^2) = 2b^4$ $b^2(-3b) = -3b^3$ $b^2(6) = 6b^2$ $(-5)(2b^2) = -10b^2$ $(-5)(-3b) = 15b$ $(-5)(6) = -30$
2.	Combine like terms.	$6b^2 - 10b^2 = -4b^2$
3.	Put it all together.	$2b^4 - 3b^3 - 4b^2 + 15b - 30$

QUICK NOTE: This problem isn't more difficult. It just has more work to do. Don't associate more work with more difficulty. That adds to math anxiety and we are trying to get rid of that.

And one last one just for fun!

EXAMPLE 7

Simplify. $(4x^2 + 3xy - 2y^2)(2x - 3y)$

SOLUTION TO EXAMPLE 7

1.	Distribute the term outside the parentheses to all the terms inside the parentheses.	$4x^2(2x) = 8x^3$ $4x^2(-3y) = -12x^2y$ $3xy(2x) = 6x^2y$ $3xy(-3y) = -9xy^2$ $(-2y^2)(2x) = -4xy^2$ $(-2y^2)(-3y) = 6y^3$
2.	Combine like terms.	$-12x^2y + 6x^2y = -6x^2y$ $-9xy^2 + (-4xy^2) = -13xy^2$
3.	Put it all together.	$8x^3 - 6x^2y - 13xy^2 + 6y^3$

TRY THESE – Multiplying Polynomials

1. $4x(3x^3 + 2x^2 - 1)$

2. $(2x - 7)(3x - 2)$

3. $(2x^2 - 1)(2x^4 - 3x - 1)$

4. $(3x + 5)(x^4 - 5)$

5. $(7x - 4)(y^2 + 4)$

6. $4xy(2xy^2 + xy - 1)$

7. $(2x^5 + 2x)(5x^4 - 4x + x^8)$

8. $4x(2x^2 + 2x - 4)$