Chapter 12: Squares and Square Roots

The phrase "squared" when dealing with exponents actually does come from the geometric shape. To find the area of a square, you would multiply the base by its height. In a square, the base and the height are the same length. So the term "squared" refers to multiplying a number by itself.

Perfect Squares

For those that don't remember perfect squares, I have included a chart to refresh your memory.

n	Multiplication	Square expression	Perfect square value
1	1×1	1^2	1
2	2×2	2^2	4
3	3×3	3^2	9
4	4×4	4^2	16
5	5×5	5^2	25
6	6×6	6^2	36
7	7×7	7^2	49
8	8×8	8^2	64
9	9×9	9^2	81
10	10×10	10^2	100
11	11×11	11^2	121
12	12×12	12^2	144
13	13×13	13^2	169
14	14×14	14^2	196
15	15×15	15^2	225

So when we multiply a whole number by itself, we get a **perfect square** value.

Square Roots

A square root is the value we get when we work our way backwards from the perfect square value. If we break down the term "square root", the word "square" refers to the perfect square number. The word "root" refers to the number we start with, n. The square root of 4 is 2. In mathematical symbols it will look like this: "$\sqrt{4} = 2$".

For Future Reference: If you think back to multiplication, a positive number times a positive number equals a positive number. A negative number times a negative number equals a positive number. So when we take the square root of a number, we actually get two roots, a positive and a negative.

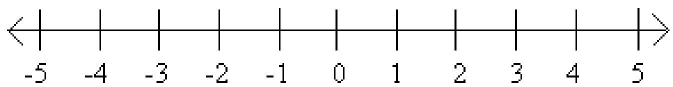
We know that $\sqrt{36} = 6$ since $6 \times 6 = 36$. But $(-6) \times (-6) = 36$ also. So $\sqrt{36} = -6$ is also a solution. Therefore, $\sqrt{36} = 6 \; and - 6$. Usually we only use the positive square root. But your math classes might also ask for the negative square root value.

If we use a perfect square value, we get a whole number for our square root. That's great! But what happens when we are not given a perfect square value? Since we don't have a calculator to give us the decimal we want, we have to learn how to estimate a square value.

This is part of a problem from the practice math competency exam.

EXAMPLE 1
Place the following number in the appropriate place on the number line.

$$\sqrt{15}$$



SOLUTION TO EXAMPLE 1
If we look at our table of perfect squares, 15 is not on the table. But we should notice that 15 is found between 9 and 16 (which are perfect squares). That means that $\sqrt{15}$ must be found in between $\sqrt{9}$ and $\sqrt{16}$. With me so far? From our chart, $\sqrt{9} = 3$ and $\sqrt{16} = 4$. So putting our logic all together, $\sqrt{15}$ has to be in between 3 and 4 on the number line.

Since 15 is closer to 16, $\sqrt{15}$ is closer to 4. When we place our point on the number line, we should place it closer to 4.

TRY THESE – Estimating Square Roots

1. $\sqrt{51}$
2. $\sqrt{121}$
3. $\sqrt{49}$
4. $\sqrt{5}$
5. $\sqrt{99}$
6. $\sqrt{34}$
7. $\sqrt{150}$
8. $\sqrt{\frac{16}{25}}$

<u>**NOTE**</u>: Notice that there is a fraction in Problem 8. Hopefully you didn't freak out at the sight. But there is one helpful property that you would need to know. If you ever have a fraction in the square root, you can to the square root of its numerator and place it over the square root of its denominator.

$$\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$$

Go back and try Problem 8 if you haven't already done so.