CHAPTER 11: GRAPHING LINEAR EQUATIONS

You will be asked to graph several problems on the math competency exam. But remember that it is all multiple choice. So you will have to be able to pick the best distractor. So here are a few pieces of information you may need when working with graphs.

Slope

Slope is the steepness of a line, which just means how fast does it move up or move down. It is a ratio of its rise over its run. It can also be described as the change in the y-direction over the change in the x-direction.

$$\text{slope} = \frac{rise}{run} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{\Delta y}{\Delta x}$$

That's a lot of symbols to know, but they all mean the same thing.

A positive slope means that the line is increasing as we move from left to right on the coordinate plane.
A negative slope means that the line is decreasing (going down) as we move from left to right on the coordinate plane.
A zero slope is a horizontal line and is given by the equation $y =$"some number"
An undefined slope is a vertical line and is given by the equation $x =$"some number".

The equations that involve perfectly horizontal and vertical lines are quite simple. There is not much to do. It would be helpful to memorize what these types of lines generally look like and if it is horizontal or vertical.

I thing that you will need to find is the slope of a line if you are given only two points. To do this we use the formula $\text{slope} = \frac{y_2 - y_1}{x_2 - x_1}$. Because we have to points, we can't simply say y-coordinate or the x-coordinate. You wouldn't know from which point to take the value. That's what those little 1's and 2's are. y_2 mean the y-coordinate from the 2nd point. So y_1 must mean the y-coordinate from the 1st point. Similarly x_2 mean the x-coordinate from the 2nd point. And x_1 means the x-coordinate from the 1st point. It does not matter which point you select as the 1st point and 2nd point. You just cannot change your mind in the middle of the problem.

EXAMPLE 1

Find the slope between the points (3, 5) and (6, 3).

SOLUTION TO EXAMPLE 1

1.	I usually place the variables above the points so I know exactly which variable goes with the number.	(x_1, y_1) $(3, 5)$ (x_2, y_2) $(6, 3)$
2.	Plug the numbers into the formula.	$\frac{3-5}{6-3}$
3.	Simplify the expression. Keep it as a fraction since slope is often written as the ratio.	$\frac{-2}{3}$

It might be helpful to use parentheses when plugging in your numbers. It might seem silly if you have all positive number in your two ordered pairs, but once you start getting negative numbers in the pairs, it is extremely easy to screw up.

If you are working a problem and get a zero in the denominator, the slope is undefined. If you get a number other than zero in the denominator and get a zero in the numerator, the slope is zero.

Slope-Intercept Form

Slope-intercept form is one way that we can write a linear equation. Slope-intercept form is given by the equation $y = mx + b$. Notice the "y" variable is by itself. That means in problems we might have to solve for "y" if it is not by itself.

Why are we making a big deal about slope-intercept form? It is because slope-intercept form gives us two important pieces of the graph of the linear equation. It tells us the slope of the line and also the **y-intercept** (the point where the line crosses over the y-axis).

The slope is represented by the variable "m" and the y-intercept is the point $(0, b)$.

So let's use the slope intercept form.

EXAMPLE 2

Find the slope of the following line: $2x + 4y = 10$

SOLUTION TO EXAMPLE 2

1.	Let's solve for the variable "y". If you need a refresher, look back in Example 4 of Chapter 6.	$2x + 4y = 10$ $2x - 2x + 4y = -2x + 10$ $4y = -2x + 10$ $\frac{4y}{4} = \frac{-2x}{4} + \frac{10}{4}$ $y = \frac{-1}{2}x + \frac{5}{2}$
2.	Now that "y" is by itself, we can find our slope and y-intercept. Slope is the number multiplied by x.	Slope $is -\frac{1}{2}$
3.	The y-coordinate of the y-intercept is "b".	y-intercept is $\left(0, \frac{5}{2}\right)$

Let's do one more "Slope-intercept Form" problem.

EXAMPLE 3

Find the slope of the following line: $3x + 7y = 14$

SOLUTION TO EXAMPLE 3

1.	Let's solve for the variable "y". If you need a refresher, look back in Example 4 of Chapter 6.	$3x + 7y = 14$ $3x - 3x + 7y = -3x + 14$ $7y = -3x + 14$ $\frac{7y}{7} = \frac{-3x}{7} + \frac{14}{7}$ $y = \frac{-3}{7}x + 2$
2.	Now that "y" is by itself, we can find our slope and y-intercept. Slope is the number multiplied by x.	Slope $is -\frac{3}{7}$
3.	The y-coordinate of the y-intercept is "b".	y-intercept is $(0,2)$

TRY THESE – Finding Slope and y-Intercept

1. Find the slope between the points $(-3, 4)$ and $(-2, -5)$

2. Find the slope between the points $(4, 6)$ and $(4, -2)$

3. Find the slope between the points $(5, 2)$ and $(-1, 2)$

4. Find the slope between the points $(5, 9)$ and $(-5, 8)$

5. Find the slope and y-intercept of the line $3y = 3x - 9$

6. Find the slope and y-intercept of the line $-4x - 3y = 12$

7. Find the slope and y-intercept of the line $x = 12$

8. Find the slope and y-intercept of the line $3x - y = -4$

Graphing Linear Equations

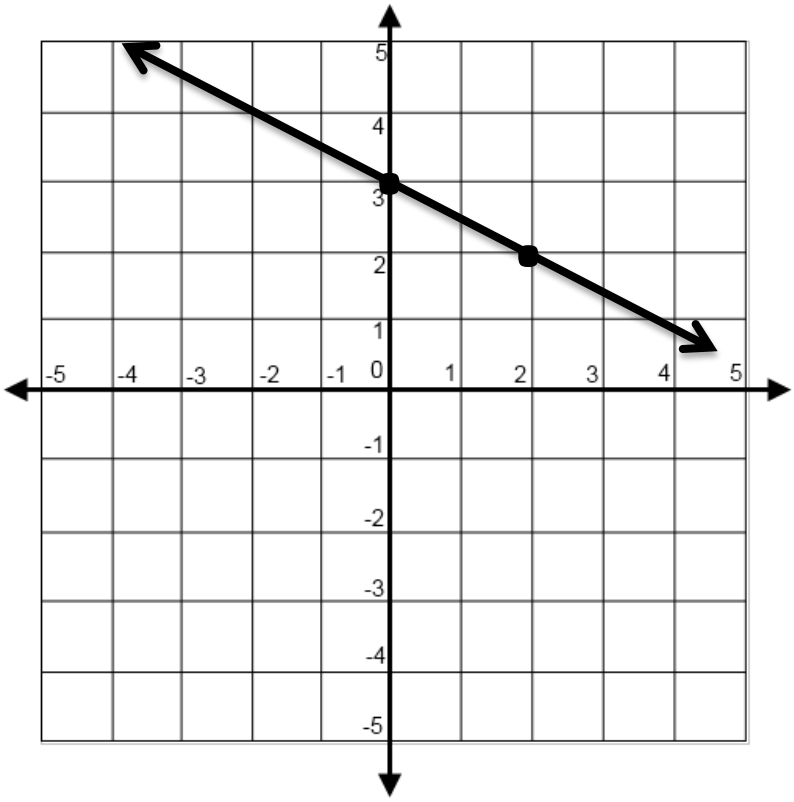
There are many different approaches you can take to graphing a linear equation. In order to graph a line, you have to have two points. Slope-Intercept Form is not always the easiest method to do, but it works almost every single time. So we need to make sure we solve our linear equation for the "y" variable. This will give us the slope of our line and the y-intercept (one of our two points that we need). As we go through the next example, I will walk you through how to get the second point.

EXAMPLE 4

Graph the line $2x + 4y = 12$

SOLUTION TO EXAMPLE 3

1.	Let's solve for the variable "y". If you need a refresher, look back in Example 4 of Chapter 6.	$2x + 4y = 12$ $2x - 2x + 4y = -2x + 12$ $4y = -2x + 12$ $\frac{4y}{4} = \frac{-2x}{4} + \frac{12}{4}$ $y = \frac{-1}{2}x + 3$
2.	Now that "y" is by itself, we can find our slope and y-intercept. Slope is the number multiplied by x.	Slope $is \frac{-1}{2}$
3.	The y-coordinate of the y-intercept is "b".	y-intercept is $(0,3)$
4.	So now that we have the slope and y-intercept, let's move to our graph.	



5.	First plot the y-intercept. Find the origin and count up that many spaces. Put a point there.
6.	Remember that slope is the rise over the run. So the top number tell us how far to move either up or down. Since the number is a negative one, we move one space down.
7.	The run is a positive two so we would move the point two space to the right.
8.	Draw your second point.
9.	Last thing we need to do is connect the points.

TRY THESE – Graphing Linear Equations

Given the linear equation, find the slope and y-intercept. Then graph on a coordinate plane.

1. $y = -3x + 1$

2. $x = 5$

3. $2x - 5y = 10$

4. $y = -2$

5. $y = \frac{1}{2}x - 3$

6. $4x + 4y = 8$

7. $x = -1$

8. $3x - 7y = 7$