Gas molecules collide with the walls, a gazillion times a second. Each collision imparts a small force for a small time on the walls. Averaged over gazillions of collisions per second, this adds up to a pressure. $f = \Delta(mv)/\Delta t$, $P=F/A$
The ideal gas: elastic tiny spherical particles

Elastic, hence the kinetic energy is conserved

Temperature is defined as this total kinetic energy of the atoms with some coefficient (to measure it in degrees).

Then, if we know the number of atoms, we can find the average kinetic energy per atom, and therefore the average speed and magnitude of $mv$ per atom.

$$F = \frac{\Delta(mv)}{\Delta t}$$

The result for the pressure is:

$$P = \frac{N}{V} kT$$

where $k$ is a universal constant

$N$ is a number of molecules

$V$ is a volume

$T$ is a temperature
The ideal gas law:

Pressure \( \uparrow \) if

Temperature \( \uparrow \) (molecules move faster)

Volume \( \downarrow \) (molecules have less space, therefore hit walls more often)

Number \( \uparrow \) (more molecules will hit walls more often)

\[ P = \frac{N}{V} kT \quad \text{or} \quad P = D_N kT \]
Atmospheric pressure

Normal pressure at sea level is:

1 atm = 14.7 psi = 101,293 Pa = 760 mm Hg

\[ 101293 \text{ Pa} = D_{\text{Hg}}gh = 13,600 \times 9.8 \times h = 133280 \times h \]

\[ h = \frac{101293}{133280} = 0.760 \text{ m} = 760 \text{ mm} \]
Mercury... what about water? How high the water will rise?

1 atm = 101293 Pa = D g h = 1,000 × 9.8 × h  
\[ h = \frac{101,293}{1,000}/9.8 = 10.34 \text{ m} \]

10.34 m = 33.9 ft

It is not possible to pull water higher than 10.34 m at normal atmospheric pressure!

Can trees be higher than 10.34 m?
Air pressure decreases with altitude.
QUESTION:
You have container filled with water. Does the fluid pressure only act on the bottom of the container?

1. Yes, since pressure is due to the weight of the water.
2. No, since water is a fluid and it can spread, the walls are needed to contain it therefore pressure also acts on the walls.
Pascal’s principle

- Pressure forces act not only on the bottom of the container, but on all sides of the container.
- If you “choose” a tiny cube of water at some depth, the pressure forces on all of its faces will be the same regardless of the cube’s orientation. The cube therefore will be at rest - hydrostatic.
- Moreover, if you divide these forces by the areas of faces, you will get the pressure on this depth: on the same level, the pressure is constant “in all directions”.

The last statement manifests Pascal’s principle.
PASCAL’S PRINCIPLE

- If an incompressible liquid is enclosed, pressure will be transmitted undiminished to all parts of the liquid and the walls of the container.

- In this figure, the pressure is much larger than $Dgh$ due to the weight of the liquid itself!
A hydraulic press

- A hydraulic lever
- The work done by the two forces is equal provided the fluid is incompressible
Questions:
Why does wood float on water and rocks sink?
Why does wood not float in air?
Why do helium balloons rise but (cold) air balloons sink?
Why do hot air balloons rise?

\[ p_T = Dgh_T \implies F_T = p_T A = Dgh_T A \]
\[ p_B = Dgh_B \implies F_B = p_B A = Dgh_B A \]
\[ F_{buoyancy} = F_B - F_T = Dgh_B A - Dgh_T A \]
\[ = Dg(h_B - h_T)A = DgHA \]
\[ = DgV = M_{\text{liquid}}g = F_G \text{ liquid} \]

\[ F_{buoyancy} = F_G \text{ displaced liquid} \]
ARCHIMEDES’ PRINCIPLE

A fluid (or gas) in the gravitational field exerts an upward force on an object equal to the weight of the fluid displaced by the object.

This force is called the **buoyant force**.

\[ F_b = D_{\text{fluid}} \cdot V \cdot g \]
Archimedes example:

A golden? Crown

When we weigh the crown in air its weight is 6.000 lb, when we weigh it submerged in water its weight is 5.689 lb. Is the crown made of gold?

\[
W_{\text{submerged}} = W_{\text{out}} - D_w g V \\
V = \left( W_{\text{out}} - W_{\text{submerged}} \right) / g / D_w \\
D = W_{\text{out}} / g / V = W_{\text{out}} / \left( W_{\text{out}} - W_{\text{submerged}} \right) D_w = \frac{6}{6 - 5.689} \times 1 \frac{g}{cm^3} = 19.293 \frac{g}{cm^3} \\

\text{The density of gold is } 19.3 \frac{g}{cm^3} \]
Archimedes problem about a crown - the way to determine the density of an object

\[ D = D_{\text{liquid}} \left( \frac{W_{\text{out}}}{W_{\text{out}} - W_{\text{submerged}}} \right) = D_{\text{liquid}} \frac{W_{\text{out}}}{\Delta W} \]

Floating objects - what is the condition?
• Example: what does it mean to float? “A boat is floating.” The boat’s mass is 50 tons. How much water is displaced?

Exactly 50 tons. The buoyancy force must be equal to the force of gravity of the boat for the boat to be static.

• If there is no equality, there must be acceleration:

A 2 liter He balloon ma = D V g - mg, therefore it flies upwards if you let go. If you do not, then T + mg = D V g

The tension force keeps it tied.

• If the average density of an object is less than the density of liquid, it floats – it rises to the surface to displace less liquid to make gravitational and buoyancy forces equal
What if the fluid is moving?
Does the fluid pressure change?

**QUESTION:**

*How does the pressure in the more narrow pipe compare to the pressure in the wider pipe?*

1. It will be higher.
2. It will be lower.
3. It will be the same.
Strangely enough, the pressure in the narrower section is LOWER than in the wider section!

**BERNOULLI'S PRINCIPLE:**

For a fluid undergoing steady flow, the pressure is lower where the fluid is flowing faster.

\[ P + Dgh + \frac{1}{2} Dv^2 = \text{constant} \]
Examples of Bernoulli principle:

Magnus effect: lift force due to pressure difference below and above the rotating cylinder (or ball) in air (water) flow.

The same principle causes the lift force in under-sonic airplanes:

The speed “above” the wing is on the average larger higher than “below”.
MECHANICS OF MATTER:
Application of laws of mechanics to systems that have many interacting point-like particles.

**Pressure** = Force per area

**Density** = m/V

Pressure in liquids: \( P = \frac{F}{A} = D \cdot h \cdot g \)

Pascal’s principle: in fluid or gas the pressure forces are equal in all directions

Archimedes’s principle: the buoyancy force \( F_b = D_f \cdot g \cdot V \)

Bernoulli’s principle:
The higher \( v \), the lower \( P \)

Pressure in gas: momentum transfer to walls

\[
P = \frac{N}{V} kT \quad \text{or} \quad P = D_N kT
\]
Temperature and Heat

- Heat - a substance ("calorium")?
  More or less heat - more or less calorium. If a hot object touches a cold object, the calorium is transferred to the cold object and it heats up.
  Substance cannot be just created and cannot just vanish.

- Count Rumford () disproved this just by watching drilling of copper cannons. His deduction was that what is transferred cannot be a substance, only motion!

- There is one more detail in his proof which I will comment on later.

- I will present the current understanding of this issue without much historical detail, because the history of "heat" is one of the longest.
What is temperature?

If measured in °K (Kelvin) it is a measure for the average KE of atoms and molecules in an object.

Example: single-atomic ideal gas

\[ \frac{3}{2} kT = \frac{1}{2} m \langle v^2 \rangle \]
Since Kelvin scale is tied to the average kinetic energy of atoms (molecules), it cannot be negative. It is called the absolute scale of temperature. The step, 1°C, is chosen for convenience.

Celcius scale: 0° melting point of water ice, 100° - boiling point of water at atmospheric pressure.

Fahrenheit scale: 32° melting point of water ice, 212° - boiling point of water at atmospheric pressure.

Conversions:

\[ ^\circ C = \frac{5}{9}(^\circ F - 32) \]

\[ ^\circ F = \frac{9}{5} ^\circ C + 32 \]

\[ ^\circ K = 273.15 + ^\circ C \]

<table>
<thead>
<tr>
<th>°F</th>
<th>°C</th>
</tr>
</thead>
<tbody>
<tr>
<td>-40</td>
<td>-40</td>
</tr>
<tr>
<td>0</td>
<td>-18</td>
</tr>
<tr>
<td>32</td>
<td>0</td>
</tr>
<tr>
<td>50</td>
<td>10</td>
</tr>
<tr>
<td>68</td>
<td>20</td>
</tr>
<tr>
<td>86</td>
<td>30</td>
</tr>
<tr>
<td>212</td>
<td>100</td>
</tr>
</tbody>
</table>
**QUESTION:**

Match the following situations with their respective temperatures:

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>Water Boiling</td>
</tr>
<tr>
<td>2.</td>
<td>Water Freezing</td>
</tr>
<tr>
<td>3.</td>
<td>A hot summer day</td>
</tr>
<tr>
<td>4.</td>
<td>A cold winter night</td>
</tr>
<tr>
<td>1.</td>
<td>100 ºC</td>
</tr>
<tr>
<td>2.</td>
<td>212 ºF</td>
</tr>
<tr>
<td>3.</td>
<td>90 ºF</td>
</tr>
<tr>
<td>4.</td>
<td>32 ºC</td>
</tr>
<tr>
<td>5.</td>
<td>100 ºC</td>
</tr>
<tr>
<td>6.</td>
<td>32 ºF</td>
</tr>
<tr>
<td>7.</td>
<td>0 ºC</td>
</tr>
<tr>
<td>8.</td>
<td>0 ºF</td>
</tr>
<tr>
<td>9.</td>
<td>-18 ºC</td>
</tr>
</tbody>
</table>
Temperature

- The temperature of an object, solid, liquid, or gaseous tells how hot or cold it is.
- When we measure the temperature, we take a small calibrated probe (a thermometer) and bring its temperature to the temperature of the object.
- Regardless of the phase of matter and the scale of a thermometer, the temperature is related to the kinetic energy of all atoms or molecules of this object.
- We will consider only a few effects related to the temperature. In solids and fluids, it is a so called thermal expansion. We will discuss gases in a little more detail.
THERMAL EXPANSION

When solids or liquids are heated they tend to expand.

The change in length of a heated solid rod is given by:

\[ \Delta L = \alpha L \Delta T \]

Change in length \quad \uparrow \quad Coefficient of linear expansion \quad \downarrow \quad Change in temperature

\[ \text{Length of a rod} \]

The law of linear thermal expansion is applied to all dimensions of an object - its width, thickness also increase with the same coefficient.

Both the area and volume increase linearly as well.

Table 5.2 Some Coefficients of Linear Expansion

<table>
<thead>
<tr>
<th>Solid</th>
<th>( \alpha \times 10^{-6}/^\circ\text{C} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Aluminum</td>
<td>25</td>
</tr>
<tr>
<td>Brass or bronze</td>
<td>19</td>
</tr>
<tr>
<td>Brick</td>
<td>9</td>
</tr>
<tr>
<td>Copper</td>
<td>17</td>
</tr>
<tr>
<td>Glass (plate)</td>
<td>9</td>
</tr>
<tr>
<td>Glass (Pyrex)</td>
<td>3</td>
</tr>
<tr>
<td>Ice</td>
<td>51</td>
</tr>
<tr>
<td>Iron or steel</td>
<td>12</td>
</tr>
<tr>
<td>Lead</td>
<td>29</td>
</tr>
<tr>
<td>Quartz (fused)</td>
<td>0.4</td>
</tr>
<tr>
<td>Silver</td>
<td>19</td>
</tr>
</tbody>
</table>
$\Delta L = \alpha L \Delta T$

**EXAMPLE:**

A 1m long rod of Aluminum is heated from 20 °C to 300 °C. How much does it expand?

$\Delta L = \alpha L \Delta T = 25 \times 10^{-6}/\degree C \times 1 \text{ m} \times 280 \degree C = 7 \times 10^{-3} \text{ m}$
JOINTS IN A BRIDGE

Which picture was taken on a hot, which on a cold day?
LIQUIDS:
Usually expand when heated which means that their volume increases.

Exception: Water, which *contracts* when heated from 0°C to 4°C.

GASES:
Gases follow the ideal gas law: \( P \times V = k \times N \times T \)

\[ V = \frac{kNT}{P} - \text{at a constant pressure a given portion of a gas expands as the temperature increases}\]
How does a hot air balloon work?
Pressure has to stay constant – it’s in equilibrium with the atmosphere.

Volume is fixed (size of ‘bag’)

Temperature is increased.

What happens to the number of molecules?

\[ PV = N kT \quad \Rightarrow \quad N = \frac{PV}{kT} \]

It decreases as temperature increases

Then: \( Mg = N m_{\text{molecule}} g \)

But the buoyancy force that just depends on the displaced volume of the cold air and its density does not change! - The balloon takes off!
What happens to the gas when the piston is depressed?

1. Its pressure increases and temperature decreases
2. Its pressure increases and temperature does not change
3. Both the pressure & temperature increase
4. Both the pressure & temperature decrease
5. Not enough information to tell
The work done by the gas

To approach the answer to the above question, we can analyze the situation from the outside:

- To compress the gas, we have to apply a force and apply it while the piston is moving – the force is directed in the direction of the piston’s displacement, hence the work done by this force is positive.

- On the other hand, the gas responds with a pressure force on the piston – equal and opposite according to the third Newton’s law. This force – the pressure force due to atomic collisions with the (moving) piston – does negative work as the gas is compressed.

- And, at last, if you let the piston go, the gas will expand pressing on a piston and now doing positive work.

- If the gas expands it does positive work, if it is compressed it does negative work.
When we introduced different phases of matter, we said that the defining features of phases are distances between atoms and extents of their interactions.

Imagine that atoms are tiny objects connected with springs. In the solid and liquid phases this springs are contracted all the time and the atoms are somehow oscillate on these springs. They have kinetic energies as much as they are moving, and those springs have potential energies as they are contracted or stretched. In the gas phase, you can also think in terms of such springs and then the potential energy inside the gas unless the gas is ideal.

The sum of the kinetic and potential energies of all atoms or molecules of the object is called the internal energy. In the ideal gas, the internal potential energy is zero.
Back to the piston

- We did some positive work
- Results?
  - The internal energy of the gas (just the temperature if it is ideal) might increase
  - The walls of the cylinder may have been heated up.
- But we do not want to talk about the walls of the cylinder, because the system will become larger (then we will recall that our hands are touching a cylinder, and a cold wind has just blown in the lab...), we want to talk only about a gas under the piston pushed by an external force.
- Then we say that the work done on the gas added energy to the gas that somehow divided into change of the internal energy of the gas and the heat expelled from the vessel.
Let us consider at another example: a vessel with a piston is heated by a torch. What happens?

Again there are two effects:

- the internal energy of the gas increases
- The piston is moving up – the gas does positive work
- Now we can spell it: the energy in the form of heat supplied to the gas and this resulted in the increase of the internal energy of the gas and in a positive work done by the gas.

\[ Q = \Delta U + W \]
First law of thermodynamics

- Now we can return to the pushed piston and respell our statement keeping only gas in the picture:
  The negative work done by the gas increased(?) its internal energy and expelled some heat from the system.

\[ \Delta U + W \]

- The energy in the form of heat added (+) or expelled from (-) the gas goes into increase (+) or decrease (-) of the internal energy of the gas and the work done by the gas
- This law defines the heat and manifests energy conservation in thermodynamics.
Again the piston

- Well, can we answer the question now? Piston is pushed \( P/T = kN/V, \ V \) decreases. \( Q = \Delta U + W \)
- What is \( \Delta U \)? \( \Delta U = Q - W \) but can we determine its sign can we tell what is larger \( Q \) or \( W \)?
- We can think in terms of how we can push the piston
  - Push very very slowly permanent thermal exchange - \( T \) stays constant, \( \Delta U=0, P \) increases (isothermal process)
  - Push very very fast: no thermal exchange: \( Q=0, T \) increases, \( P \) increases (adiabatic process)
  - Everything between these two extremes, but what about these two? Cannot yet tell.

1. Pressure increases and temperature decreases
2. Both the pressure & temperature decrease