Monkey shot

Assume the zookeeper shoots off the banana the same time the monkey lets go. Where should the zookeeper aim to make sure the monkey gets the banana?

1. Straight at the monkey.
2. Below the monkey.
3. Above the monkey.
1. **Banana**: \( y = v \sin \theta \cdot t - \frac{1}{2} gt^2 \)

   \[
   \text{Monkey: } y = h - \frac{1}{2} gt^2
   \]

   \[
   v \sin \theta \cdot t - \frac{1}{2} gt^2 = h - \frac{1}{2} gt^2
   \]

   \[
   h = v \sin \theta \cdot t \Rightarrow \frac{h}{\sin \theta} = vt
   \]

2. **Banana**: \( y = v_{0y} t - \frac{1}{2} gt^2 \)

   \[
   \text{Monkey: } y = -\frac{1}{2} gt^2
   \]

   \[
   v_{0y} t - \frac{1}{2} gt^2 = -\frac{1}{2} gt^2
   \]

   \[
   \Downarrow
   \]

   \[
   v_{0y} = 0
   \]

*Aim straight at the monkey!*
Harmonic force

Spring Force:
\[ F = -k \Delta d \text{ (Hooke’s law)} \]

Equilibrium:
\[ ma = mg - kd_0 = 0 \]
\[ d_0 = \frac{mg}{k} \]

Top: \[ ma = k\Delta d + mg > 0 \] directed downward

Bottom:
\[ ma = mg - k\Delta d < 0 \] directed upward
FLIGHT & Newton's third law

Relative velocity of air

Force of air on wing = LIFT

Force of wing on air
Why are wings not of much use in outer space?

There is no air in outer space!
Nothing for wings to push against.

Spacecraft are completely controlled by rockets.
Gravity:

- A force associated with the mass of objects.
- Responsible for weight & free fall on earth, planetary orbits, and tides.
- Acts between all massive objects.
- Gravitational force is CENTRAL - is directed from one object to the other.
Newton’s Law of Gravity:

The force of gravity is acting between any two objects that have a property of mass.

This force is proportional to the masses of the objects and inversely proportional to the distance squared between them.

This force is attractive - acts on each object along the line connecting the two objects in the direction of the other object

\[ F = G \frac{m_1 m_2}{R^2} \]

\[ G = 6.67 \times 10^{-11} \text{ N m}^2/\text{kg}^2 \] - a universal constant
Inertial and gravitational mass

- We just introduced quantitatively two different masses
- Inertial mass - measure of inertia and determined by comparing accelerations of different particles provided forces acting on them are the same
- Gravitational mass - mass, that changes the space around it by the gravitational field and acting with a force of gravity on any other particle that has this property of mass, a mass in the Newton’s law of gravity.
- Are these masses the same?
  - Experimentally - yes they are
- Is this a coincidence?
  - Newtonian mechanics - yes
  - General relativity - no, there is a deep reason for them to be the same
Gravitational field

- Any object that has a property of mass changes the space around it: causes the gravitational field, so that the force of gravity is acting on any other massive object (object with a mass) put in this field.

- The earth’s gravitational field:

\[ F_G = mg = G \frac{Mm}{R^2} \]

\[ g = \frac{F_G}{m} = G \frac{M}{R^2} \]

In principle, we can accept a notion of a gravitational field as an independent quantity. If an object with a mass \( m \) is places in a field \( g \), the force of gravity acting on the object is \( mg \). So we can measure the field by measuring the force of gravity and the mass.
Example:

What is the gravitational force between two 1000 kg masses separated by 1 cm?

\[ F = G \frac{m_1 m_2}{R^2} \]

\[ = 6.67 \times 10^{-11} \text{ N m}^2/\text{kg}^2 \times 1000\text{kg} \times 1000\text{kg}/(0.01\text{m})^2 \]

\[ = 0.667 \text{ N} \]

How much would this force accelerate one of the masses?

\[ a = \frac{F}{m} = 0.00067 \text{ m/s}^2 \]

*Gravity is a very weak force!*
Weighing the earth

- Need to measure $G$
  - Measure the force of gravity between two objects - Henry Cavendish
- Need to measure the earth’s radius
  - Eratosthenes

\[ g = G \frac{M_E}{R_E^2} \implies M_E = \frac{gR_E^2}{G} \]

\[ M_E = 9.8 \times (6.4 \times 10^6)^2 / 6.67 \times 10^{-11} = 6 \times 10^{24} \text{ kg} \]
PLANETARY ORBITS

Newton’s “Thought Experiment”:

Throw a ball from a mountaintop faster and faster (neglecting air resistance). What happens?
Orbits:

• Small speed: \( F_G > \frac{mv^2}{r} \), Object falls

• As speed increases, \( F_G = \frac{mv^2}{r} \), gravitational force is just enough to keep object on circular orbit.

• If speed is further increased, object can fly off - depends on total energy (escape velocity).
NOTE: In general, orbits are ellipses, not circles. The Newton’s law summarizes three laws discovered by Johannes Kepler’s. We will comment on them later on.
EXAMPLE:

How fast would the space shuttle 400 km off the surface of the earth have to go to stay in a circular orbit?

\[ F_G = G \frac{m \, M_E}{R^2} = m \frac{v^2}{R} \quad \Rightarrow \quad v^2 = \frac{G \, M_E}{R} \]

\[ M_E = 5.979 \times 10^{24} \text{ kg} \]

\[ R = R_E + 400 \text{ km} = 6376 \text{ km} + 400 \text{ km} = 6.776 \times 10^6 \text{ m} \]

\[ v^2 = 5.885 \times 10^7 \quad \Rightarrow \quad v = 7,672 \text{ m/s} = 17,165 \text{ mph} \]
Example: The radius of the Moon is equal to 0.273 or the Earth’s radius. The mass of the Moon is equal to 0.012 that of the Earth. Find “g” on the Moon surface.

\[
F = G \frac{M_E m}{R_E^2} = mg_E \quad \Rightarrow \quad g_E = G \frac{M_E}{R_E^2}
\]

\[
g_M \frac{R_M^2}{g_E R_E^2} = \frac{M_M}{M_E} \quad \Rightarrow \quad g_M = \frac{M_M}{M_E} \left(\frac{R_E}{R_M}\right)^2
\]

\[
\frac{g_M}{g_E} = \frac{0.012}{0.273^2} = 0.16 \quad \Rightarrow \quad g_M = 9.81 \cdot 0.16 = 1.57 \frac{m}{s^2}
\]