Dynamics

What causes motion?
What causes changes in motion?
- Mass
- Inertia
- Momentum
- Force
MASS

Mass is:

- A measure for the amount of matter contained in an object
- We will further introduce two more meanings or definitions of mass - they would follow from the physical laws

Measurement? - comparison

- A glass of water is less massive than a bucket of water

Mass is not associated with any direction:

- a scalar quantity
- the amount of matter cannot be negative: mass is always positive

Unit of mass: 1 kg - mass of 1 liter of distilled water
Inertia

- Inertia is a reluctance to change (the state of motion)...
- “- innate force of matter, the power of resisting by which every body as much as in it lies, perseveres in its state, either of rest, or of uniform motion, in a straight line.”

What does it depend?
- Mass: the larger the mass is, the more inertia the object has - (a car and a truck...)

The “amount of motion” or linear momentum - a product of mass of a particle by its velocity, $p=mv$ - also a vector directed in the same direction as the velocity.

Intuitively, something that has a large momentum is harder to stop.
Newton's FIRST LAW:

Aristotle: “the moving force stops when the force propelling it stops its action”
Galileo - Newton: “

“An object will remain at rest or in motion with constant velocity unless acted on by a net external force”

In absence of external agent, the motion of an object does not change

Newton’s amount of motion = \( m \cdot v \) (magnitude and direction)
Newton’s first law – inertia law: comments

- Cannot be derived from any experiment !! In Nature, there is always some resistance, some force that is changing motion. Ultimately - an idealized law.

- If the above problem is resolved, how can we verify this law? If different observers see a particle moving, each of them will measure $v' = v + v_O$, where $v_O$ is a velocity of a particular observer. All of them will find the motion of a particle to be uniform (they measure $v'$ and $v_O$ and have to agree about $v$) only if all measured velocities are constant! Therefore, the Newton’s first law says that

- There exist such frames of reference (the inertial frames of reference) in which a particle “perseveres in its state of moving uniformly (or resting, $v=0$) in a straight line, as far as it is not compelled to change that state by external forces impressed upon it.”
Inertial frames of reference

- If an observer’s frame accelerates with respect to the inertial frame, the Newton’s first law is not valid in this frame.
  Example: Consider a toy car on the floor of a stationary train or a train moving uniformly – nobody touches it – it’s at rest with respect to the floor, however it is likely to start moving with respect to the train if it accelerates (starts motion, brakes, etc.)

- All natural systems of reference may be inertial only approximately – the Earth is rotating and the stars are as well. Hence, this part of the law is idealized as well.

- Nevertheless, the first Newton’s law can be verified to the enough extent – when the limitations do not hurt measurements (reasonably small velocities, reasonably short observation times, etc.)
Newton’s SECOND LAW:

The change of motion is proportional to the net external force acting on a particle. The direction of this change is the same as the direction of the applied net force.

\[
\Delta(mv) = F\Delta t \quad \text{change of motion is proportional to the force}
\]

\[
\frac{\Delta(mv)}{\Delta t} = F \quad \text{rate of momentum change is equal to the force}
\]

If the mass does not change, \[
\frac{\Delta(mv)}{\Delta t} = m \frac{\Delta v}{\Delta t} = ma = F
\]

\[
a = \frac{F}{m} \quad \text{acceleration is directly proportional to the force and inversely proportional to the mass}
\]

\[
m = \frac{F}{a} \quad \text{mass is the measure of inertia - how large a force should be to change an acceleration by one unit?}
\]
Newton’s second law: comments

- Since acceleration is a vector, a force is a vector quantity – magnitude and direction
- Mass is a “measure of inertia” the more massive the object is, the more difficult to accelerate it, the more force is needed – one way of comparing (measuring) masses
- In any inertial frame of reference, an observer will measure the same acceleration \((a=a')\) and, since the mass is independent of the frame of reference, the same force.
- The force is always directed in the same direction as the velocity change or acceleration, but not necessarily the velocity itself.
- The second Newton’s law relates the velocity change with the force, not the velocity with the force.
What is force?

An agent acting on a body (action exerted on a body) that causes an acceleration or a distortion of the body.

Examples:

1. Pushing or pulling
2. Weight
3. Tension
4. Reaction of the surface
5. Friction
6. Elastic (spring) force
7. Gravity
“... proportional to the NET EXTERNAL”

What do these mean?

**Net**: Force is a vector quantity. If several forces act on the same body, the “net” force is the vector sum of all the forces.

\[ F_1 + F_2 = 0 \]
Another example...

Thrust from wing motion

Force due to air currents

Force due to air resistance

Gravity

Vector addition:

Net Force
How are forces measured?

Usually, the unknown forces are “equilibrated” or compensated by the known forces such that the net force and therefore the acceleration is zero.

Unit is determined from $F=ma$

Unit of force 1 Newton = $1 \text{ kg}\times\text{m/s}^2$

1 Newton ≈ Weight of an English (rather small) apple

English units: Pound

1 lb = 4.45 N
External:

To change its motion, the force has to act on the object from the outside (external agent).

Example: Cannot push your car from inside the car.

It is trivial if we recall that we restrict our objects to material points - there is no “inside of a point”!!

Now, let’s consider a system of two points:

- from the inside, these points may interact - act on each other with some forces,

- from the outside, however, they can be observed as one (point-like object) - e.g. a system of Earth-Moon revolving around the Sun

- Then from the outside, the first and second Newton’s laws have to be applicable.
The Third Newton’s law

- If there is no net external force acting on this pair of particles in some inertial frame, they (as a couple) have to move uniformly.
- However, they may interact between each other – act on each other with forces...
- The sum of these forces, however, must be zero! Otherwise, it would bring about a change in the overall uniform motion.
- If the parts of the system interact with each other, the forces they exert on each other are equal by magnitude and opposite to each other, so that their (vector) sum is zero - the third Newton’s law.
Newton’s THIRD Law:

When one object exerts a force on a second object, the second exerts an equal and opposite force on the first. - Note: both objects are the two parts of a closed system!

Within a closed system, the total $mv$ is conserved, the change of $mv$ in one part has to be compensated by the change in the other!
Various forces:

Force of **Gravity**

If you release an object of mass $m$, it falls on the ground. If the air resistance is negligible, the acceleration is $g$ (downwards).

Therefore, the gravitational force

$$F_G = mg$$

Directed downwards (to the center of the Earth)

We will return to the force of gravity, one of the fundamental forces, later.
How to measure the force of gravity?

- The effect: a point with a mass \( m \) falls freely if acted on by the force of gravity only.
- Compensate the force of gravity - do not let the point to fall - support it.
- Support? Then the point interacts with a support.
- Two ways of support - one is to hang it (by the string) – then the elastic force of the string (rope, spring, etc.) will compensate the “pull of gravity”.

\[
\text{Gravity} = mg
\]

\[
\text{Pull on the string} = mg = \text{weight!}
\]

\[
\text{Tension force} = -mg
\]
Weight

- Gravity pulls a block down ($=mg$ on the block)
- A block pulls the string down ($=mg$ on the string) - weight
- A string pulls back with tension ($=-mg$ on the block) 3rd Newton’s Law
- As a result, the net force on the block is $mg-mg=0$, block is at rest. Net force acting on an object is calculated by summing all forces acting on this object.
- What do we measure? THE WEIGHT
- The weight is the force acting on the support that prevents a body from falling.

$\text{Gravity} = mg$

Pull on the string = $mg$ = weight!

Tension force = $mg$
The second way is to put it on the floor scales

- Gravity pulls a block down (=mg on the block)
- A block pushes on the floor scales down (=mg on the scales) - weight
- The scales pushes up with a surface reaction force(=-mg on the block) 3rd Newton’s Law
- As a result, the net force on the block is mg-mg=0, block is at rest
- What do we measure? THE WEIGHT
- The weight is the force acting on the support that prevents a body from falling.

Push on scales=mg

Gravity =mg

Surface Reaction force =mg
Mass, Weight and the Force of Gravity

- Mass is a quantity of matter, a measure of its inertia, a scalar quantity independent of the frame of reference.
- Weight is a force acting on an object to prevent it from falling due to gravity.
- In an inertial frame of reference, Weight = mg and directed downward.
- The force of gravity is equal to mg (regardless of the frame), directed to the center of the earth. Caused by the Earth.
- \( g = 9.8 \text{ m/s}^2 \) close to the surface of the earth; it varies a little bit from place to place, it depends on the altitude and is quite different on different planets, moons, stars...
- On the Moon: the force of gravity and weight in an inertial frame will change because "g" is different, but not mass!
Weight in accelerating frames

- Consider a person standing on a floor scale in an elevator.
- Elevator moves at a constant velocity up, down, or zero. Then the frame related to the elevator is inertial. The person is at rest: Gravity (-mg) + Reaction of scales (mg) = Net force (0) = ma => a=0. The scales show the magnitude of the pushing force (=mg) - weight
- Elevator moves at a constant acceleration, a, upward. Then the frame related to the elevator is not inertial. The person is at rest with respect to the elevator: Gravity (-mg) + Reaction of scales (N) = Net force = ma

- mg + N = ma, => N = mg + ma

The scales show the magnitude of the pushing force (=mg+ma>mg) - weight is larger than mg!!
**Weightlessness**

- Elevator moves at a constant acceleration, $a$, downward. Then the frame related to the elevator is not inertial. The person is at rest with respect to the elevator: Gravity ($-mg$) + Reaction of scales ($N$) = Net force = $-ma$
  
  $-mg + N = -ma$, $\Rightarrow N = mg - ma$

The scales show the magnitude of the pushing force ($=mg - ma > mg$) - weight is smaller than $mg$!!

- How much smaller can the reaction of the surface and, therefore, the weight, be? When $a=g$, $N = mg - ma = mg - mg = 0$, weight is equal to zero - weightlessness!! No contact - a person is falling freely under action of one force - gravity.

- If the downward, acceleration increases further, there is still no force to act on a person - no contact with a scale - weight is still zero, the elevator moves faster than the falling person until the person hits the ceiling!
Example:

How much would a 1400 kg car accelerate if a net force of 280 Newtons acts on it? How long would it take to speed it up to 10 m/s?

\[ F = m \ a \Rightarrow F / m = m \ a / m \Rightarrow a = F / m \]

\[ a = 280 \text{ N}/1400 \text{ kg} = 0.2 \text{ m/s}^2 \]

\[ a = \Delta v / \Delta t \Rightarrow \Delta t = \Delta v / a = (10 \text{ m/s}) / (0.2 \text{ m/s}^2) = 50 \text{ s} \]
Mass on string experiment:

QUESTION:

What will happen if I pull the string fast (Case 1)?
What will happen if I pull the string slowly (Case 2)?

In case 1 the lower string will break, in case 2 the upper.

Why?

Case 1: the lower string stretches, the load does not move - the tension is too large - the string breaks

Case 2: the load accelerates and pulls on the upper string...
Force Pairs:

Ground pushes on chair
Chair pushes on ground

Dog pushes on chair
Chair reacts on dog

Earth pulls on dog
Dog pulls on earth
How can a locomotive pull a wagon if the wagon pulls back at the locomotive with the same force as the locomotive pulls forward?
In order to figure out if an object is moving, we need to look at the net force on that object.

The “opposite forces”

1. Have the same nature
2. Act on different objects (that are parts of the system).
1. The pair of forces acts on different objects – if we want to know if an object accelerates, we need to look only at the forces acting on that object.

2. In the case of the locomotive, we need to look at the force from the tracks versus the force from the wagon. If the force from the tracks is larger than the force from the wagon, the locomotive will accelerate.
Consider a train, a locomotive and two wagons:

Given its acceleration is \( a = 1 \text{ m/s}^2 \) and masses of wagons are 10,000 kg each, what is the tension in the connections?

Blue wagon: \( ma = T_B \)

\[ T_B = 10,000 \times 1 = 10,000 \text{ N} \]

Green wagon: \( ma = T_G - T_B \), \( T_G = ma + T_B \)

\[ T_G = 10,000 + 10,000 = 20,000 \text{ N} \]

The longer the train, the larger the tension in the first spring.
Elastic forces:

- The force of pressure (on the ground) - because my legs are elastic and do not bend because of the body push.

- The force of reaction of the ground - because the floor is elastic and does not let you fall through.

- The tension of the string, spring, etc. - elastic forces do not let them break under the pull (or push) of a load.

- The nature of all elastic forces is electromagnetic - in this chapter we just introduce them as example of forces and explore their macroscopic action. In general,

\[
\text{Elastic force} = - \text{coefficient} \times \text{stress}
\]

Where the stress is proportional to some relative distortion in the subject - The elastic force can act on a particle, but it cannot be initiated by a particle - one cannot distort a particle - need at least two, then \( \Delta d/d... \)
Friction = Force of resistance to relative motion between two bodies of substances in physical contact

- Nature is Electromagnetic: caused by some distortion in the material caused by a moving (or even not moving) particle
- The friction force can act on a particle, but it cannot be initiated by a particle - one cannot distort a particle

**Static friction** (Friction force acting on the object at rest with respect to the surface)

**Kinetic friction** (Friction force acting on the object moving with respect to the surface)

How to measure the friction force, and what is it equal to?
Static and kinetic friction

- In principle, the friction force “wants” to compensate the pulling or pushing force, so if the latter is zero, the friction force is zero too, if the pulling (pushing) force increases, the friction increases as well.
- But there is a limit - the maximum force of friction, and when it is achieved, the friction does not increase with the further increasing pulling force.
- Friction force is independent of the area of contact.

Friction force = pulling force - no motion - static friction

Friction force < pulling force - acceleration - kinetic friction
Maximum friction force

- Maximum friction force depends on properties of the surface(s) and the force of reaction of the surface;
- No contact (N=0) - no friction
- $f_{\text{Max}} = \mu \times N$
- $\mu$ - friction coefficient that depends on properties of the surface(s)
- $\mu_{\text{Kinetic}} < \mu_{\text{Static}}$ - maximum static friction is

![Diagram showing friction force vs. pulling force](image)
Centripetal force

- For the object to move at a constant speed around a circle, a centripetal acceleration is required, \( a_c = \frac{v^2}{r} \)
- Therefore, the force is required \( F = ma_c = \frac{mv^2}{r} \)
- There is **NO** such force in nature, the **NET** force must provide the centripetal force.

Examples:

1. if you swing a mass on a string around your hand, the tension force is the net, centripetal force.
2. On the circular orbit the force of gravity, \( Mg \), is the net, centripetal force.
QUESTION:

A car is moving around a bend with a radius of 100 meters at 25 m/s. What force does the road have to exert on the tires to keep the car on the road?

Friction force keeps the car at rest (not sliding).

Maximum friction force = \( \mu N = \mu mg \geq F_c = \frac{mv^2}{r} \)

\[ \mu mg \geq \frac{mv^2}{r} \implies \mu g \geq \frac{v^2}{r} \implies \mu \geq \frac{v^2}{gr} \]

\[ \mu \geq \frac{25^2}{9.8 \cdot 100} = 0.64 \]

If the road is dry, the friction coefficient can be as large as 0.64, but if it is wet, it is less and you have to slow down!
Forces and Motion

Different forces cause different motion (Newton’s 2nd law)

Some examples:

- Object at rest or at constant velocity: No net force (Newton’s first law), object is in *equilibrium*.

- Constant acceleration: Constant force: Example: *Free Fall*

- *Projectile motion*: Free Fall + horizontal uniform motion.

- *Simple harmonic motion*, due to a special case of a Force that is *not* constant.
Free Fall: Motion of an object on which only the constant force of gravity acts.

QUESTION:
I am throwing a rock straight up in the air. Which of the following statements are correct?

1. The acceleration is positive as it goes up and negative as it goes down.
2. The acceleration is always pointing downwards.
3. At the highest point the velocity and acceleration will be zero.
4. At the highest point the acceleration is unchanged, but the velocity is zero for an instant.
Now, assume you are throwing a rock at an angle...

Initial velocity

What happens to the velocity?
How does the rock move?
Force is $F = mg$, straight down.

Acceleration is $g$, straight down.

Since acceleration points straight down at all times, only the vertical part of the velocity will be affected.

The horizontal part stays constant, since there is no force in that direction!

Initial velocity
So what happens?

PROJECTILE MOTION: PARABOLA